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THE ALTERATION OF PROFILE ANALYSIS TO
ACCOMMODATE TESTING FUNCTIONS

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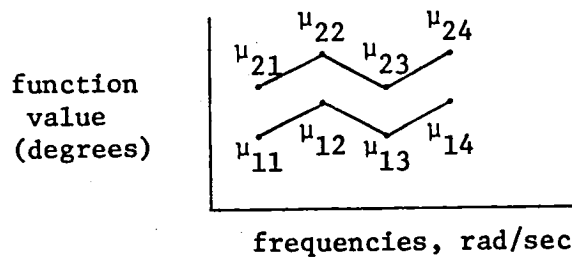
National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

THE ALTERATION OF PROFILE ANALYSIS TO ACCOMMODATE TESTING FUNCTIONS

The purpose of this task is to develop a methodology for testing for differences between several pilot describing functions, where the data points represent averages at various frequencies. Typically a function represents a treatment or treatment combinations which are to be compared. Data are taken in replicates and an output is measured for each frequency. It is not unusual to experience 4-8 functions representing a like number of experimental conditions in a real-time piloted aircraft simulation. Typically, as many as 8-16 frequencies may be involved.

Consider the following plot for the special case of two functions.



Here, the μ_{ij} represents the population mean for the j^{th} frequency under the i^{th} function. The problem is to determine whether or not, simultaneously, the means for function 1 are equal to the means for function 2, across frequencies. The extension to more than two functions is also of considerable interest.

Specifically the goals of the task are as follows:

- (i) Develop and describe the methodology for testing for differences between functions.

(ii) Determine how to approach the problem of assessing the power of the test. Supply charts or tables for power.

(iii) Use the power results to recommend how to design such experiments; for example determine an effective number of frequencies; in addition, what is the maximum number of functions that will allow a reasonably sensitive test. Also what is a reasonable number of replicates?

(iv) Discuss software considerations.

Basic Assumptions for the Experiment

In this section we shall discuss the distributional and data structure proposed for the test on k functions ($k \geq 2$). Consider initially two functions, in the context of Figure I. Suppose that we have two random vectors

$$\underline{x}_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1t} \end{bmatrix} \quad \text{and} \quad \underline{x}_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2t} \end{bmatrix}$$

where x_{ip} is the scalar output for the i^{th} function and the p^{th} frequency. These are basic output measurements. It is assumed that each of the vectors follows a multivariate normal distribution with common variance-covariance matrix Σ , the latter a $t \times t$ matrix. The practical implication here is that within each function the observations are correlated and the

correlation structure is the same for each of the two functions. For the first function n_1 independent vectors are taken and for the second function n_2 vectors are observed.

The extension to more than two functions is obvious. It is assumed that there are k -multidimensional vectors $\underline{x}_1, \underline{x}_2, \dots, \underline{x}_k$,

$$\begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1t} \end{bmatrix}, \quad \begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2t} \end{bmatrix} \quad \dots \quad \begin{bmatrix} x_{k1} \\ x_{k2} \\ \vdots \\ x_{kt} \end{bmatrix}$$

The assumption is made that \underline{x}_i is multivariate normal with mean $\underline{\mu}_i$ ($i=1,2,\dots,k$) and variance-covariance matrix Σ .

Hypothesis

Consider again the case of two functions illustrated in Figure I. A reasonable hypothesis that accomplishes the goals outlined here, is given by

$$H_0: \begin{bmatrix} \mu_{11} = \mu_{21} \\ \mu_{12} = \mu_{22} \\ \mu_{13} = \mu_{23} \\ \mu_{14} = \mu_{24} \end{bmatrix} \quad (1)$$

or, in terms of vectors,

$$H_0: \underline{\mu}_1 = \underline{\mu}_2,$$

where $\underline{\mu}_i$ is the mean vector for the i^{th} function. One can easily see that this is the hypothesis that the experimenter needs to test. Namely, the null hypothesis states that the mean value of the two functions are equivalent at each frequency. In the general case of k functions and t frequencies the hypothesis is

$$H_0: \underline{\mu}_1 = \underline{\mu}_2 = \underline{\mu}_3 = \dots = \underline{\mu}_k \quad (2)$$

where each vector is t -dimensional. We need a test procedure for testing the joint statement given in (2). In addition, we need guidelines on the experimental design that will adequately give evidence in favor of the alternate hypothesis when it is appropriate.

Profile Analysis

The problem posed here appears to be very similar to Profile Analysis [2] which is a multivariate technique in which a "repeated measures" design is used and the treatments are broken into groups. Basically, measurements on individuals (or individual items) are conducted for each of say t treatments, with the observations correlated from treatment to treatment. The purpose of the experiment is to determine if there are differences in treatment means. When the treatments are put into an additional classification variable called "groups" then the procedure becomes what is called Profile Analysis. Consider, again Figure I and allow the functions to take the role of the groups and frequencies take the role of the treatments. This would appear to be an application of Profile Analysis.

In a Profile Analysis the test on treatments is not valid unless no interaction exists between treatments and groups. Thus, initially a test for parallelism (no interaction) is usually conducted before the test on treatments is attempted. The hypothesis of the test on parallelism is given by

$$H_0: \begin{bmatrix} \mu_{11} - \mu_{12} = \mu_{21} - \mu_{22} \\ \mu_{13} - \mu_{12} = \mu_{23} - \mu_{22} \\ \mu_{14} - \mu_{13} = \mu_{24} - \mu_{23} \end{bmatrix}$$

The test involves a Hotelling's T^2 [2] and is quite easy to extend to more than two groups.

Alteration of Profile Analysis to Accommodate Testing Functions

One can easily see (easier to visualize in the case of two functions or groups) that if groups and interactions between groups and frequencies (functions and frequencies) are not statistically significant, one can interpret this as implying that functions are not significantly different point by point or frequency by frequency. One mode of verification of this is to note that if two functions are parallel between frequencies the only way that the two function averages can be the same is for the two functions to coincide. On the other hand, function averages might be very close together but because of non-parallelism (interaction) the two functions may be far from coincidental. Thus neither a significance test on groups (function) averages or interaction will suffice for our purposes. Rather it is the joint hypothesis of the two that is relevant.

Searching into the structure of the problem more closely suggests that the condition of coincidental functions we have described here is identical to the hypothesis in (1) or (2) for the general case. Thus it can be said that we have a profile analysis in which we are attempting to simultaneously detect parallelism and group (function) effects but we are doing it with a single hypothesis and thus a single test and not with two tests.

Test Statistic and Procedure

For the hypothesis in (1), we are simply testing the equality of two mean vectors. For the j^{th} function, n_j independent vectors (replicates) are obtained and we have two averages $\bar{\underline{x}}_1, \bar{\underline{x}}_2$ which are joint estimates of the mean vectors $\underline{\mu}_1$ and $\underline{\mu}_2$ (t -dimensional). The sample data is used to compute an estimate of the variance-covariance Σ . Thus we have for the first function, n_1 independent vectors, and n_2 independent vectors for the second function. The estimate of Σ is found by obtaining sample variances and covariances for each function and pooling over functions. Thus the estimates have $n_1 + n_2 - 2$ degrees of freedom. The test statistic is given by [2]

$$T^2 = (\bar{\underline{x}}_1 - \bar{\underline{x}}_2) S^{-1} (\bar{\underline{x}}_1 - \bar{\underline{x}}_2) \quad (3)$$

where

$$S = \hat{\Sigma} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)$$

The statistic in equation (3) follows Hotelling's T^2 distribution with $n_1 + n_2 - 2$ degrees of freedom. If the same number of replicates are taken for each function, say n , then of course $n_1 + n_2 - 2$ is replaced by $2(n-1)$. The statistic

$$\frac{(n_1+n_2-t-1)}{(n_1+n_2-2)t} T^2 \quad (4)$$

follows a F distribution with t and $n_1 + n_2 - t - 1$ degrees of freedom. This fact allows us to handle the power quite easily.

The extension to more than two functions is quite easily done. Consider the hypothesis of equation (2), which, written in expanded form is given by

$$\begin{aligned} H_0: \mu_{11} &= \mu_{21} = \mu_{31} = \dots = \mu_{k1} \\ \mu_{12} &= \mu_{22} = \mu_{32} = \dots = \mu_{k2} \\ &\vdots \\ \mu_{1t} &= \mu_{2t} = \mu_{3t} = \dots = \mu_{kt} \end{aligned}$$

Once again, there are vectors of averages $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k$, t -dimensional. This data is used to compute a pooled estimate of the variance-covariance matrix $\hat{\Sigma}$. In addition a $t \times t$ matrix (hypothesis matrix) is given by

$$h_{rs} = \sum_{j=1}^k \frac{1}{n_j} T_{rj} T_{js} - \frac{1}{N} G_r G_s .$$

Here T_{kj}^r is the sum of the data on the r^{th} frequency for function j . $N = \sum_{j=1}^k n_j$. G_r is the grand total of all observations on the r^{th} frequency. The test statistic is the largest eigenvalue of the matrix. HE^{-1} , where E is the error sum of squares matrix, which is given by

$$E = \hat{\Sigma} \cdot \text{d.f.}$$

where d.f. are the degrees of freedom for estimating the variance-covariance matrix (d.f. = $\sum_{i=1}^k n_i - k$).

Examples of the computation for both the special case and the general case will be given in a later section. Charts for the use of the largest root statistic can be found in Morrison [2]. A computer program for the computation of the test statistic will be found in the Appendix.

Power of Tests

The power associated with the case of two functions is quite easy to handle from a theoretical point of view. In a practical sense there are some difficulties that are not insurmountable. The test statistic in (3) follows F_{t, n_1+n_2-t-1} under the hypothesis of equality of means stipulated in H_0 . One can immediately notice that the denominator degrees of freedom automatically imply an experimental restriction, namely that $n_1 + n_2 > (t+1)$. The test procedure, of course, involves rejection of H_0 if the calculated test statistic in (4) exceeds the upper tail percentage point of the F-distribution. If $\mu_1 \neq \mu_2$ the test statistic follows the non central F-distribution [1] with degrees of freedom t and $n_1 + n_2 - t - 1$, and with non centrality parameter

$$\delta^2 = \frac{n_1 n_2}{n_1 + n_2} (\mu_1 - \mu_2)' \Sigma^{-1} (\mu_1 - \mu_2) \quad .$$

Thus for a specific difference $\underline{\mu}_1 - \underline{\mu}_2$ between the mean vectors and a specific variance covariance matrix, the power is given by the probability

$$\Pr[F'_{t,n_1+n_2-t-1,\delta^2} > F_{\alpha,t,n_1+n_2-t-1}]$$

where $F'_{t,n_1+n_2-t-1,\delta^2}$ is a non central F variate and F_{α,t,n_1+n_2-t-1} is the upper α^{th} percentage point of the F distribution. Thus the probability distribution of the non central F is needed. This distribution is discussed in detail in Graybill [1]. Power charts are offered in Morrison [2] and are quite easy to use. An explanation of their use is given in the text.

In order to compute the power one must know Σ , or at least have an estimate of it. For the study described here, hypothetical values for the variance-covariance matrices are used, these values generally taken from previous studies at NASA Langley. A power study is described in the following section. This study was done to arrive at appropriate values of sample sizes (number of replicates) and number of frequencies that provide good power for specific values (in terms of order of magnitude) of the differences $(\underline{\mu}_1 - \underline{\mu}_2)$ that are being detected between the functions.

The power computation for the test for more than two functions is extremely difficult and is wrought with practical difficulties that actually prohibit its use. Approximations are available [3]. However, the approximations involve many quantities that must be estimated from the data. The non central F probabilities are monotonic increasing functions of the non centrality parameter which, if one investigates the structure of δ^2 , is very pleasing. That is, δ^2 represents "how

false" the null hypothesis is before we would wish to reject H_0 with a high probability, and as δ^2 grows large we would expect the power to increase monotonically. However, the structure of the approximations of the power of the "largest root test" is such that several non centrality parameters appear and the power is not a monotonic function of them which suggests that for at least some cases the approximation is not particularly good.

It would seem reasonable then that sample sizes and other recommendations should be done for testing two functions at a time. In addition, the user can assess his own power in a particular case by computing the power through the use of tables of the non central F. This will be reasonable as long as one does not study too many functions simultaneously. This does not mean that one must restrict the number of functions in the test, but rather in a power assessment it would only be accurate if the number of functions were kept low, say four or less. It should be emphasised here that the value, t , the number of frequencies, n_1 , the number of replicates are more crucial than k , the number of functions.

Example of Test Computation

In this section we present some hypothetical data and numerical examples of the computation for cases of two functions and three functions. The output here is not what NASA Langley personnel has at its disposal but it does represent a representative type format for the output. For the case of two functions there are $t = 5$ frequencies and $n_1 = n_2 = 4$.

S T A T I S T I C A L A N A L Y S I S S Y S T E M

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1

OBS	GROUP	Y1	Y2	Y3	Y4	Y5
1	1	12.4	9.5	15.4	13.8	15.2
2	1	13.3	10.2	14.7	13.9	15.4
3	1	13.1	10.4	14.9	14.2	14.4
4	1	12.8	10.1	15.2	14.0	14.9
5	2	10.6	9.3	12.3	12.3	11.6
6	2	9.3	8.7	12.1	12.1	12.4
7	2	9.9	9.0	11.6	11.8	12.0
8	2	10.1	8.8	11.9	11.9	11.9

S T A T I S T I C A L A N A L Y S I S S Y S T E M

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ANALYSIS OF VARIANCE PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
GROUP	2	1 2

NUMBER OF OBSERVATIONS IN DATA SET = 8

STATISTICAL ANALYSIS SYSTEM
ANALYSIS OF VARIANCE PROCEDURE

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DEPENDENT VARIABLE: Y1

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	1	17.11125000	17.11125000	77.34	0.0001	0.928005	4.1125
ERROR	6	1.32750000	0.22125000				
CORRECTED TOTAL	7	18.43875000					
					0.47037219		11.43750000
							Y1 MEAN
SOURCE	DF	ANOVA SS	F VALUE	PR > F			
GROUP	1	17.11125000	77.34	0.0001			

STATISTICAL ANALYSIS SYSTEM
ANALYSIS OF VARIANCE PROCEDURE

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DEPENDENT VARIABLE: Y2

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	1	2.42000000	2.42000000	22.00	0.0034	0.785714	5.4912
ERROR	6	0.66000000	0.11000000		STD DEV		Y2 MEAN
CORRECTED TOTAL	7	3.08000000			0.33166248		9.50000000

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	1	2.42000000	22.00	0.0034

STATISTICAL ANALYSIS SYSTEM
ANALYSIS OF VARIANCE PROCEDURE

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DEPENDENT VARIABLE: Y3

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	1	18.91125000	18.91125000	203.53	0.0001	0.971364	2.2559
ERROR	6	0.55750000	0.09291667		STD DEV		Y3 MEAN
CORRECTED TOTAL	7	19.46875000			0.30482235		13.51250000

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	1	18.91125000	203.53	0.0001

S T A T I S T I C A L A N A L Y S I S S Y S T E M

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ANALYSIS OF VARIANCE PROCEDURE

DEPENDENT VARIABLE: Y4

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	1	7.60500000	7.60500000	194.17	0.0001	0.970026	1.5224
ERROR	6	0.23500000	0.03916667		STD DEV		Y4 MEAN
CORRECTED TOTAL	7	7.84000000			0.19790570		13.00000000

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	1	7.60500000	194.17	0.0001

STATISTICAL ANALYSIS SYSTEM

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ANALYSIS OF VARIANCE PROCEDURE

DEPENDENT VARIABLE: Y5

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	1	18.00000000	18.00000000	120.67	0.0001	0.952633	2.8662
ERROR	6	0.89500000	0.14916667		STD DEV		Y5 MEAN
CORRECTED TOTAL	7	18.89500000			0.38622101		13.47500000

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	1	18.00000000	120.67	0.0001

STATISTICAL ANALYSIS SYSTEM

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ANALYSIS OF VARIANCE PROCEDURE

E = ERROR SS&CP MATRIX

DF=6

	Y1	Y2	Y3	Y4	Y5
Y1	1.32750000	0.76500000	-0.22250000	0.22250000	-0.58250000
Y2	0.76500000	0.65000000	-0.21500000	0.25000000	-0.49000000
Y3	-0.22250000	-0.21500000	0.55750000	0.12750000	-0.06750000
Y4	0.22250000	0.25000000	0.12750000	0.23500000	-0.27000000
Y5	-0.58250000	-0.49000000	-0.06750000	-0.27000000	0.89500000

PARTIAL CORRELATION COEFFICIENTS FROM THE ERROR SS&CP MATRIX / PROB > IR!

DF=5	Y1	Y2	Y3	Y4	Y5
Y1	1.000000	0.817283	-0.258637	0.398363	-0.534401
	0.00000	0.0248	0.5755	0.3761	0.2166
Y2	0.817283	1.000000	-0.354441	0.634796	-0.637548
	0.0248	0.00000	0.4354	0.1256	0.1235
Y3	-0.258637	-0.354441	1.000000	0.352252	-0.095559
	0.5755	0.4354	0.00000	0.4384	0.8385
Y4	0.398363	0.634796	0.352252	1.000000	-0.588733
	0.3761	0.1256	0.4384	0.00000	0.1643
Y5	-0.534401	-0.637548	-0.095559	-0.588733	1.000000
	0.2166	0.1235	0.8385	0.1643	0.00000

STATISTICAL ANALYSIS SYSTEM

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ANALYSIS OF VARIANCE PROCEDURE

H = ANOVA SS&CP MATRIX FOR: GROUP

DF=1	Y1	Y2	Y3	Y4	Y5
Y1	17.11125000	6.43500000	17.98875000	11.40750000	17.55000000
Y2	6.43500000	2.42000000	6.76500000	4.29000000	6.60000000
Y3	17.98875000	6.76500000	18.91125000	11.99250000	18.45000000
Y4	11.40750000	4.29000000	11.99250000	7.60500000	11.70000000
Y5	17.55000000	6.60000000	18.45000000	11.70000000	18.00000000

CHARACTERISTIC ROOTS AND VECTORS OF: E INVERSE * H, WHERE H = ANOVA SS&CP MATRIX FOR: GROUP E = ERROR SS&CP MATRIX

CHARACTERISTIC ROOT	PERCENT	CHARACTERISTIC VECTOR V'EV=1				
		Y1	Y2	Y3	Y4	Y5
201.51859391	100.00	0.41787631	0.79418981	1.07215163	0.44848061	1.25686420
0.00000000	0.00	0.90392450	-4.05018675	-2.51037663	5.22322795	-0.21822003
0.00000000	0.00	0.85487543	-0.51987437	-0.28125967	0.15670677	-0.45645118
0.00000000	0.00	-0.16047073	-0.32073506	0.81093019	0.24622078	-0.71718513
0.00000000	0.00	-1.18817472	2.25265548	0.13465891	0.05632777	0.15785824

S T A T I S T I C A L A N A L Y S I S S Y S T E M

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ANALYSIS OF VARIANCE PROCEDURE

HANOVA TEST CRITERIA FOR THE HYPOTHESIS OF NO OVERALL GROUP EFFECT

H = ANOVA SS&CP MATRIX FOR: GROUP
 E = ERROR SS&CP MATRIX
 P = DEP. VARIABLES = 5
 Q = RANK OF H = 1
 NE = OF OF E = 5
 S = MIN(P,Q) = 1
 M = .5(ABS(P-Q)-1) = 1.5
 N = .5(NE-P-1) = 0.0

 HOTELLING-LAWLEY TRACE = $TR(E^{-1}H)$ = 201.51859391 (SEE PILLAI'S TABLE #3)

F APPROXIMATION = $2(S*N+1)*TR(E^{-1}H)/(S*S*(2M+S+1))$ WITH $S(2M+S+1)$ AND $2(S*N+1)$ DF

$F(5,2) = 80.01$ PROB > F = 0.0123

 PILLAI'S TRACE $V = TR(H*INV(H+E))$ = 0.99506218 (SEE PILLAI'S TABLE #2)

F APPROXIMATION = $(2N+S+1)/(2M+S+1) * V/(S-V)$ WITH $S(2M+S+1)$ AND $S(2N+S+1)$ DF

$F(5,2) = 80.01$ PROB > F = 0.0123

 WILKS' CRITERION $L = DET(E)/DET(H+E)$ = 0.00493782 (SEE BKA VOL 53 P 347)

$W = -(NE-.5(P-Q+1))*LN(L)$ = 18.5879

 ROY'S MAXIMUM ROOT CRITERION = 201.51859391 (SEE AMS VOL 31 P 625)

FIRST CANONICAL VARIABLE YIELDS AN F UPPER BOUND

$F(1,6) = 1209.11$ (UPPER BOUND)

08.53.27 03/07/78

8:53 TUESDAY, MARCH 7, 1978

1 DATA ONE; INPUT GROUP Y1-Y5;
2 CARDS;

NOTE: DATA SET WORK.ONE HAS 12 OBSERVATIONS AND 6 VARIABLES.
NOTE: THE DATA STATEMENT USED 0.43 SECONDS AND 128K.

15 PRUC PRINT;

NOTE: THE PROCEDURE PRINT USED 0.81 SECONDS AND 128K AND PRINTED PAGE 1.

15 PROC ANOVA; CLASSES GROUP;

```

15      MODEL Y1-Y5=GROUP;
16      MANOVA H=GROUP/PRINTH PRINTE;

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NOTE: THE PROCEDURE ANOVA USED 1.53 SECONDS AND 170K AND PRINTED PAGES 2 TO 10.
NOTE: SAS USED 170K MEMORY.

NOTE: BARR, GOODNIGHT, SALL AND HELWIG
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RALEIGH, N.C. 27605

PROCESSOR ENDED, COMPLETION CODE = 5000, 00000							
	CARDS	LINE	CPU SEC	EXCPS	KBS	PRIORITY	TOTAL
NUMBER OF	17	24	3	97	780	PRIORITY	KBS=1024 BYTE-SEC
CHARGES	.02	.02	.07	.05	.02	.14	.32

S T A T I S T I C A L A N A L Y S I S S Y S T E M

8:53 TUESDAY, MARCH 7, 1978

1

OBS	GROUP	Y1	Y2	Y3	Y4	Y5
1	1	12.4	9.5	15.4	13.8	15.2
2	1	13.3	10.2	14.7	13.9	15.4
3	1	13.1	10.4	14.9	14.2	14.4
4	1	12.8	10.1	15.2	14.0	14.9
5	2	10.6	9.3	12.3	12.3	11.6
6	2	9.3	8.7	12.1	12.1	12.4
7	2	9.9	9.0	11.6	11.8	12.0
8	2	10.1	8.8	11.9	11.9	11.9
9	3	14.8	12.4	17.4	16.0	14.8
10	3	14.9	11.7	17.1	16.0	14.9
11	3	15.3	11.9	18.5	16.3	15.1
12	3	15.1	12.2	16.8	15.9	15.0

↑
further

5 - *Significance*

S T A T I S T I C A L A N A L Y S I S S Y S T E M

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ANALYSIS OF VARIANCE PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
GROUP	3	1 2 3

NUMBER OF OBSERVATIONS IN DATA SET = 12

STATISTICAL ANALYSIS SYSTEM
ANALYSIS OF VARIANCE PROCEDURE

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DEPENDENT VARIABLE: Y1

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	2	51.43166667	25.71583333	156.91	0.0001	0.972121	3.2045
ERROR	9	1.47500000	0.16388889		STD DEV		Y1 MEAN
CORRECTED TOTAL	11	52.90666667			0.40483193		12.63333333

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	2	51.43166667	156.91	0.0001

STATISTICAL ANALYSIS SYSTEM

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ANALYSIS OF VARIANCE PROCEDURE

DEPENDENT VARIABLE: Y2

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	2	19.76000000	9.88000000	93.60	0.0001	0.954128	3.1391
ERROR	9	0.95000000	0.10555556		STD DEV		Y2 MEAN
CORRECTED TOTAL	11	20.71000000			0.32489514		10.35000000

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	2	19.76000000	93.60	0.0001

S T A T I S T I C A L A N A L Y S I S S Y S T E M

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ANALYSIS OF VARIANCE PROCEDURE

DEPENDENT VARIABLE: Y3

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	2	50.42166667	25.21083333	225.21	0.0001	0.980410	2.2825
ERROR	9	1.00750000	0.11194444		STD DEV		Y3 MEAN
CORRECTED TOTAL	11	51.42916667			0.33458100		14.65833333

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	2	50.42166667	225.21	0.0001

S T A T I S T I C A L A N A L Y S I S S Y S T E M

8:53 TUESDAY, MARCH 7, 1978 6

ANALYSIS OF VARIANCE PROCEDURE

DEPENDENT VARIABLE: Y4

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	2	32.41166667	16.20583333	448.78	0.0001	0.990072	1.3557
ERROR	9	0.32500000	0.03611111		STU DEV		Y4 MEAN
CORRECTED TOTAL	11	32.73666667			0.19002924		14.01666667

SOURCE	DF	ANOVA SS	F VALUE	PR > F
GROUP	2	32.41166667	448.78	0.0001

STATISTICAL ANALYSIS SYSTEM
ANALYSIS OF VARIANCE PROCEDURE

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DEPENDENT VARIABLE: Y5

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	2	23.80166667	11.90083333	113.34	0.0001	0.961813	2.3201
ERROR	9	0.94500000	0.10500000		STD DEV		Y5 MEAN
CORRECTED TOTAL	11	24.74666667			0.32403703		13.96666667
SOURCE	DF	ANOVA SS	F VALUE	PR > F			
GROUP	2	23.80166667	113.34	0.0001			

STATISTICAL ANALYSIS SYSTEM

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ANALYSIS OF VARIANCE PROCEDURE

E = ERROR SS&CP MATRIX

DF=9	Y1	Y2	Y3	Y4	Y5
Y1	1.47500000	0.70000000	-0.47750000	0.29750000	-0.49750000
Y2	0.70000000	0.95000000	-0.06500000	0.19000000	-0.54000000
Y3	-0.47750000	-0.06500000	1.00750000	0.00750000	-0.21750000
Y4	0.29750000	0.19000000	0.00750000	0.32500000	-0.23000000
Y5	-0.49750000	-0.54000000	-0.21750000	-0.23000000	0.94500000

PARTIAL CORRELATION COEFFICIENTS FROM THE ERROR SS&CP MATRIX / PRUB > !R1

DF=8	Y1	Y2	Y3	Y4	Y5
Y1	1.000000	0.591344	-0.391701	0.429584	-0.421357
	0.0000	0.0718	0.2630	0.2152	0.2252
Y2	0.591344	1.000000	-0.066440	0.341940	-0.569923
	0.0718	0.0000	0.8553	0.3335	0.0854
Y3	-0.391701	-0.066440	1.000000	0.013107	-0.222906
	0.2630	0.8553	0.0000	0.9713	0.5359
Y4	0.429584	0.341940	0.013107	1.000000	-0.415021
	0.2152	0.3335	0.9713	0.0000	0.2330
Y5	-0.421357	-0.569923	-0.222906	-0.415021	1.000000
	0.2252	0.0854	0.5359	0.2330	0.0000

STATISTICAL ANALYSIS SYSTEM

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9

ANALYSIS OF VARIANCE PROCEDURE

H = ANOVA SS&CP MATRIX FOR: GROUP

DF=2	Y1	Y2	Y3	Y4	Y5
Y1	51.43166667	30.83000000	50.87416667	40.58583333	31.66083333
Y2	30.83000000	19.76000000	30.14000000	25.03000000	16.63000000
Y3	50.87416667	30.14000000	50.42166667	39.95083333	31.97083333
Y4	40.58583333	25.03000000	39.95083333	32.41166667	23.69666667
Y5	31.66083333	16.63000000	31.97083333	23.69666667	23.80166667

CHARACTERISTIC ROOTS AND VECTORS OF: E INVERSE * H, WHERE H = ANOVA SS&CP MATRIX FOR: GROUP E = ERROR SS&CP MATRIX

CHARACTERISTIC ROOT	PERCENT	CHARACTERISTIC VECTOR V'LV=1				
		Y1	Y2	Y3	Y4	Y5
450.21043656	90.63	0.54500665	0.31402293	0.84238821	0.89104337	1.10467913
6.25629712	1.37	-0.59176348	0.48946933	-0.27940395	1.18298042	-0.53870690
0.00000000	0.00	-0.07407905	1.07117428	0.28708306	-1.21733547	0.17646957
0.00000000	0.00	0.26891350	-0.61165940	0.80057146	-0.42590350	-0.58166402
0.00000000	0.00	0.91375709	-0.38162881	-0.03212027	-0.54292847	-0.36515582

STATISTICAL ANALYSIS SYSTEM

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ANALYSIS OF VARIANCE PROCEDURE

MANOVA TEST CRITERIA FOR THE HYPOTHESIS OF NO OVERALL GROUP EFFECT

Parameter described in

*portion for testing overall
ANOVA*

H = ANOVA SS&CP MATRIX FOR: GROUP
E = ERROR SS&CP MATRIX
P = DEP. VARIABLES
U = RANK OF H
NE = DF OF E
S = MIN(P,U)
M = .5(ABS(P-U)-1)
N = .5(NE-P-1)

1.000000
1.5

2(10,12) = 6.1

HOTELLING-LAWLEY TRACE = $TR(E^{-1}H)$ = 456.46673368 (SEE PILLAI'S TABLE #3)
 F APPROXIMATION = $2(S*N+1)*TR(E^{-1}H)/(S*S*(2M+S+1))$ WITH $S(2M+S+1)$ AND $2(S*N+1)$ DF
 $F(10,8) = 182.59$ PROB > F = 0.0001

PILLAI'S TRACE $V = TR(H*INV(H+E))$ = 1.85997240 (SEE PILLAI'S TABLE #2)
 F APPROXIMATION = $(2N+S+1)/(2M+S+1) * V/(S-V)$ WITH $S(2M+S+1)$ AND $S(2N+S+1)$ DF
 $F(10,12) = 15.94$ PROB > F = 0.0001

WILKS' CRITERION $L = DET(E)/DET(H+E)$ = 0.00030543 (SEE BKA VOL 53 P 347)
 $\lambda = -(NE-.5(P-U+1))*LN(L)$ = 56.6566

ROY'S MAXIMUM ROOT CRITERION = 450.21043656 (SEE AMS VOL 31 P 625)
 FIRST CANONICAL VARIABLE YIELDS AN F UPPER BOUND
 $F(2,9) = 2025.95$ (UPPER BOUND)

See Chart in summary

Significant at 0.01 level

First, all univariate F-statistics are given comparing function values for separate frequencies. In addition, the $\hat{\Sigma}$ and H matrices are given. The Hotelling's T^2 value is given for the case of two functions and the largest eigenvalue of $E^{-1}H$ is given for the case of three functions. In both cases, the test statistic is significant. These examples are shown in the Appendix.

Recommendation of Experimental Design Through Power Study

An extensive Power Study was made in which the power of the test was computed for varying number of frequencies, number of replicates, correlation structure, and assigned differences between the function means. A set of plots are provided in order to give the reader an indication of what magnitude of power can be expected for various combinations of the parameters and to devise recommendations of what are reasonable values for the number of replicates and frequencies, given a particular correlation structure. It should be emphasized that regardless of power considerations there is a certain restriction that must hold. For the case of two functions, if we call n the number of replicates for each frequency-function combination, the denominator degrees of freedom for the F statistic will be non-positive unless

$$2n > t - 1 \quad (5)$$

Thus equation (5) represents a necessary restriction in the case of two functions. In the general case it would be advisable to keep the same

restriction, i.e., that given by equation (5) because it is unlikely that the quality of the test (i.e., power) will be good at all unless n satisfies equation (5). In addition since it is recommended to compute the power on the basis of Hotelling's T^2 (tests on two functions) one cannot assess the power unless equation (5) holds.

We attempt here to give some rationale regarding our power study. The most difficult variable to cope with was the covariance structure. Keep in mind that the correlations represent association between observations from one frequency to another within functions. The following covariance or correlation structures were studied:

- (a) Low correlation, correlation constant (0.1, 0.2)
- (b) High correlation, (0.6-0.9)
- (c) Mixed correlation between high and low (0.1-0.95) .

In addition, the power (through the noncentrality parameter) is a function of $\mu_1 - \mu_2$, the difference between the means of the two functions. Here, we are assuming that there is a typical difference which is a proportion of σ , where σ is the same at each frequency. Proportions of 0.3, 0.5, 0.7, and 1.0 are used in this study. We feel that it is reasonable to assume that $0.7\sigma - 1.0\sigma$ is a reasonable range in the difference between the two functions which one would like to detect. It is clear from the plots that for any value less than 0.7σ , experimentation is much too demanding if one wishes to obtain high power. The significance level of the test was fixed at $\alpha = 0.10$. Plots of power against frequency are shown in the Appendix for various correlation structures. Curves are shown for various proportions of σ which one is attempting to detect.

The purpose of the plots is to attempt to show the role of t , the number of frequencies and n , the number of replicates, on the power. The plots indicate the following:

(a) All correlations equal - low correlations (0.1, 0.2). The role of t , the number of frequencies is displayed dramatically in the plots shown. As t grows large, the power is reduced, and for $t > 10$ the only condition under which power is moderately good is when $n > 10$. Thus one can assume that the test will be ineffective unless $t < 10$; but if t must exceed 10, the number of replicates should be 10-12.

(b) High correlation (0.7-0.9). Here it is very clear that despite the number of frequencies, in order to obtain any effectiveness in the test, at least 12 replicates should be used. Of course, it is probably an unusual experimental situation for which all correlations would be high.

(c) Correlations mixed between high and low. This condition likely represents the most typical experimental condition that one confronts. Again, however, it appears as if small numbers of replicates will not give power values that are acceptable if one desires a quality test. For 8-10 frequencies, $n = 10$ is barely acceptable whereas for $t > 10$, a sample size of at least 12 is necessary.

Conclusions

A test procedure is given for testing equality of k different pilot describing functions. The method for handling the power for the case of two functions is described. It is felt that this is not an unreasonable

assessment of power as long as the number of functions is four or less. For rather typical types of correlation structures a power study is made in order to determine reasonable number of replicates and frequencies. Two other recommendations come to light, in addition to those involving the experimental design. These are given in the following two paragraphs.

In using the techniques described here the NASA Langley personnel should be very careful to sample to determine what correlation category prevails. The power is very much dependent on the correlation. In fact, if possible it might be advisable to take data initially to determine the approximate correlation structure, then supplement with the required data according to the experimental design recommendations.

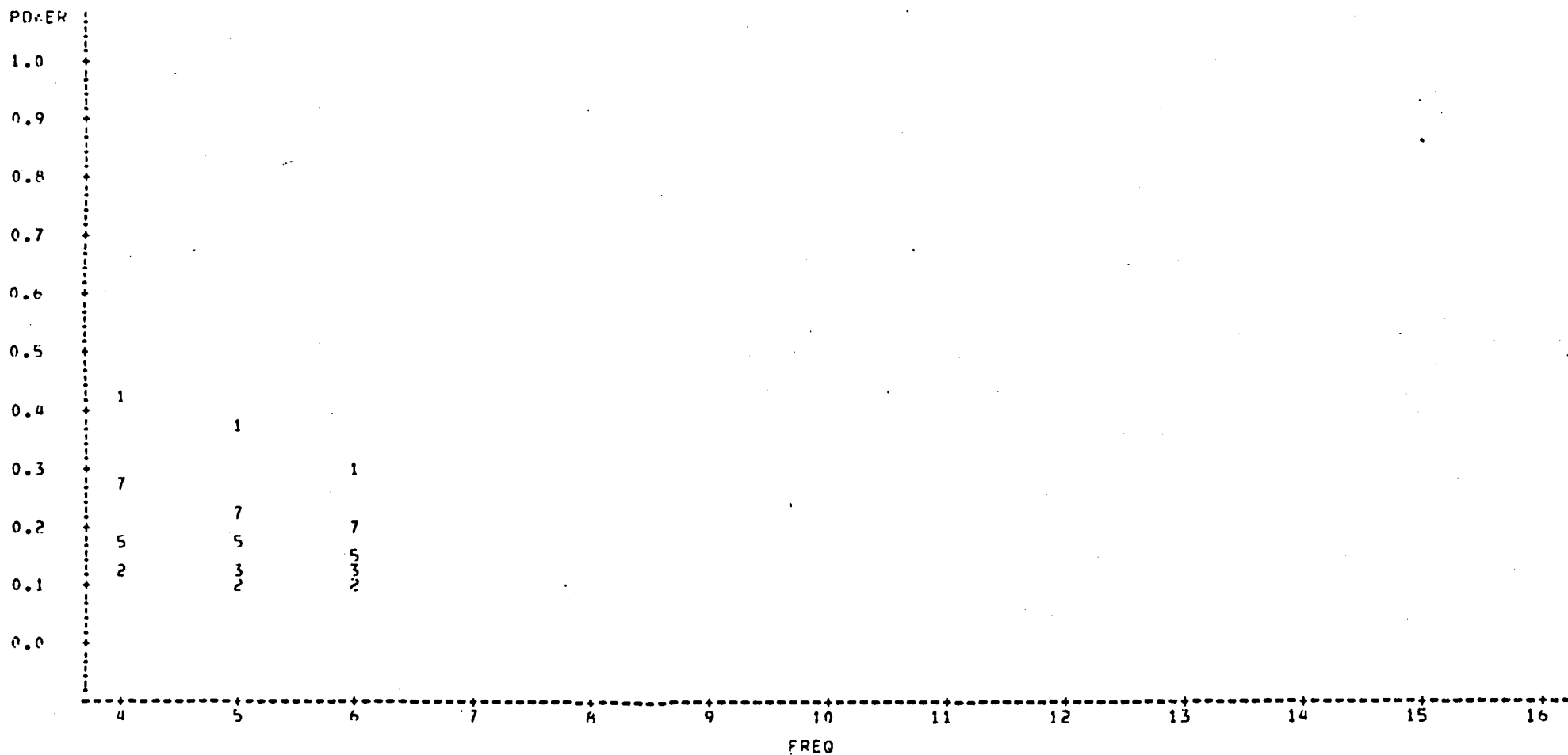
The user should not be attached to a particular significance level. We used $\alpha = 0.10$ for the power study. However, because of the inherent lack of power in tests such as these one should be prepared to consider that a test which is significant at the 0.15 or even 0.20 level is evidence in favor of differences between functions. This is particularly true if one is forced to use a large number of frequencies.

CORRELATION = .1

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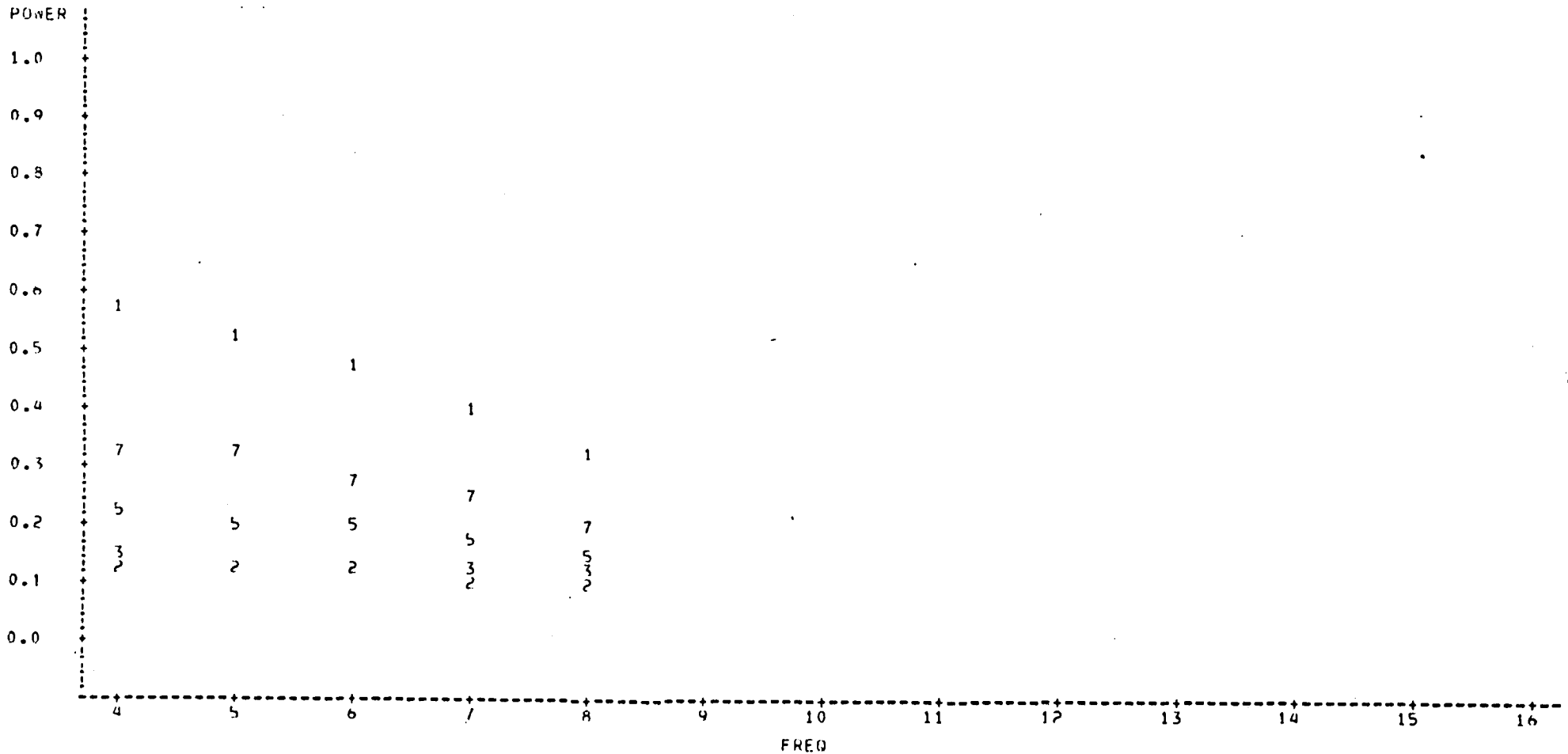


NOTE: 1 OBS HIDDEN

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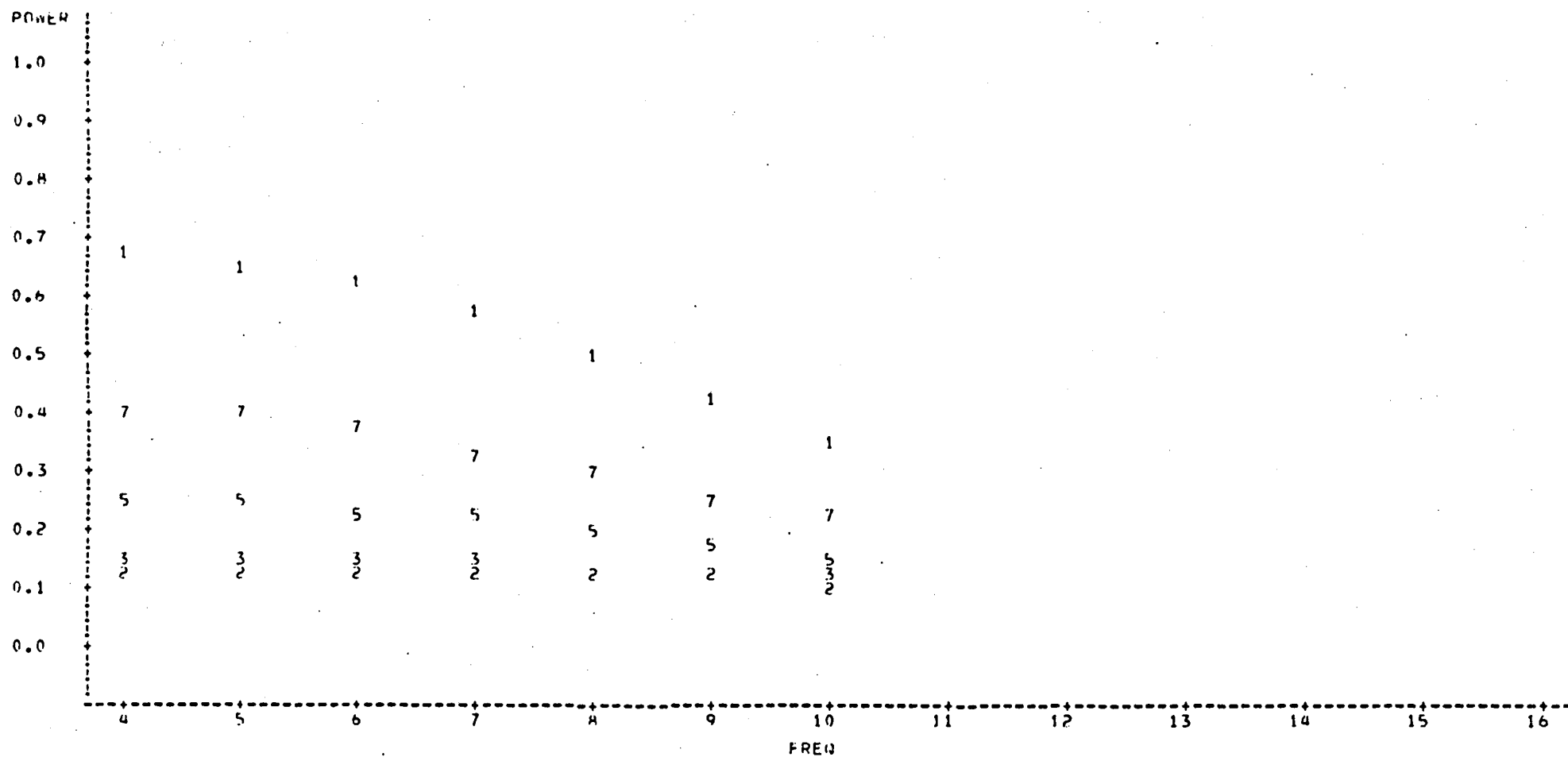


NOTE: 2 ORS HIDDEN

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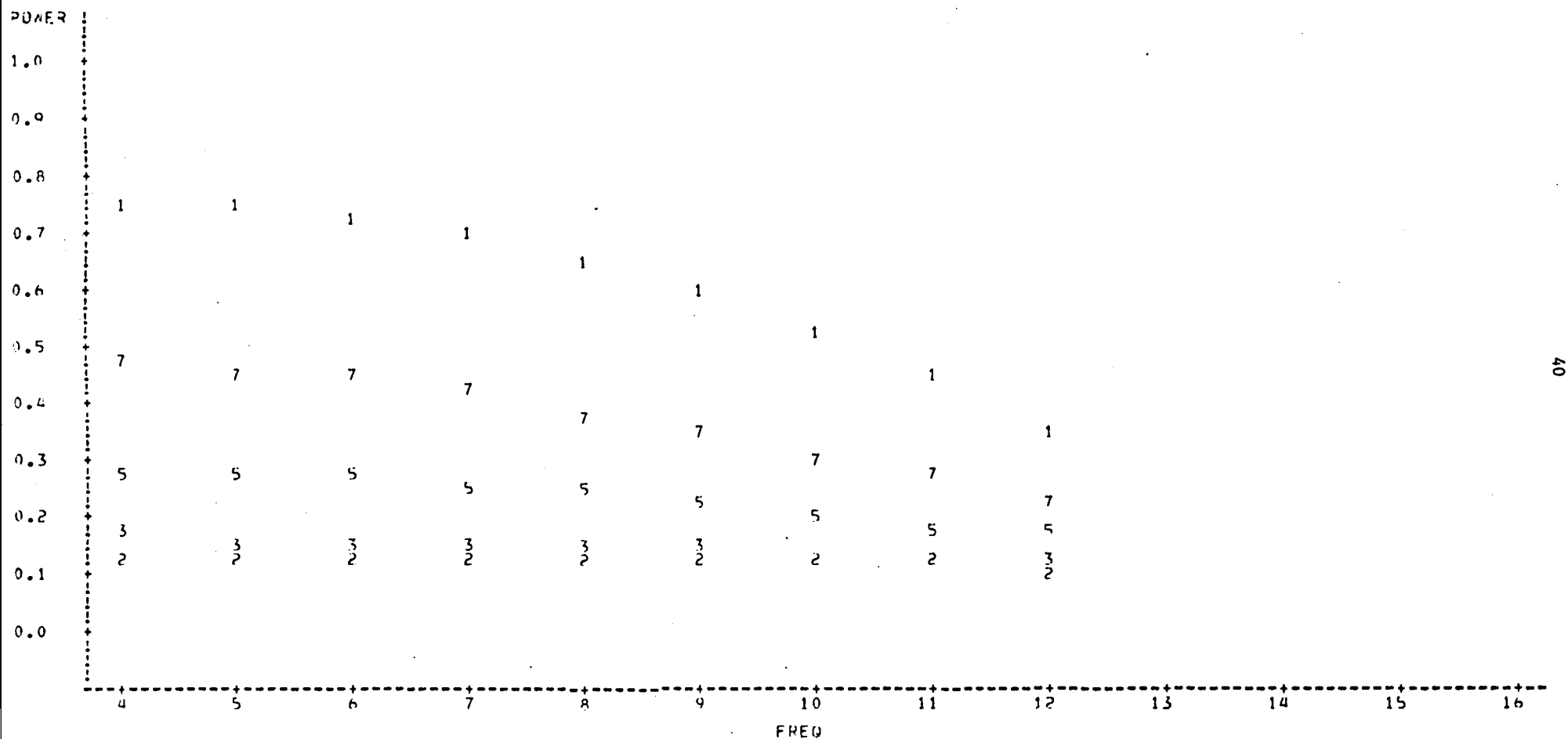
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NOTE: 2 DMS HIDDEN

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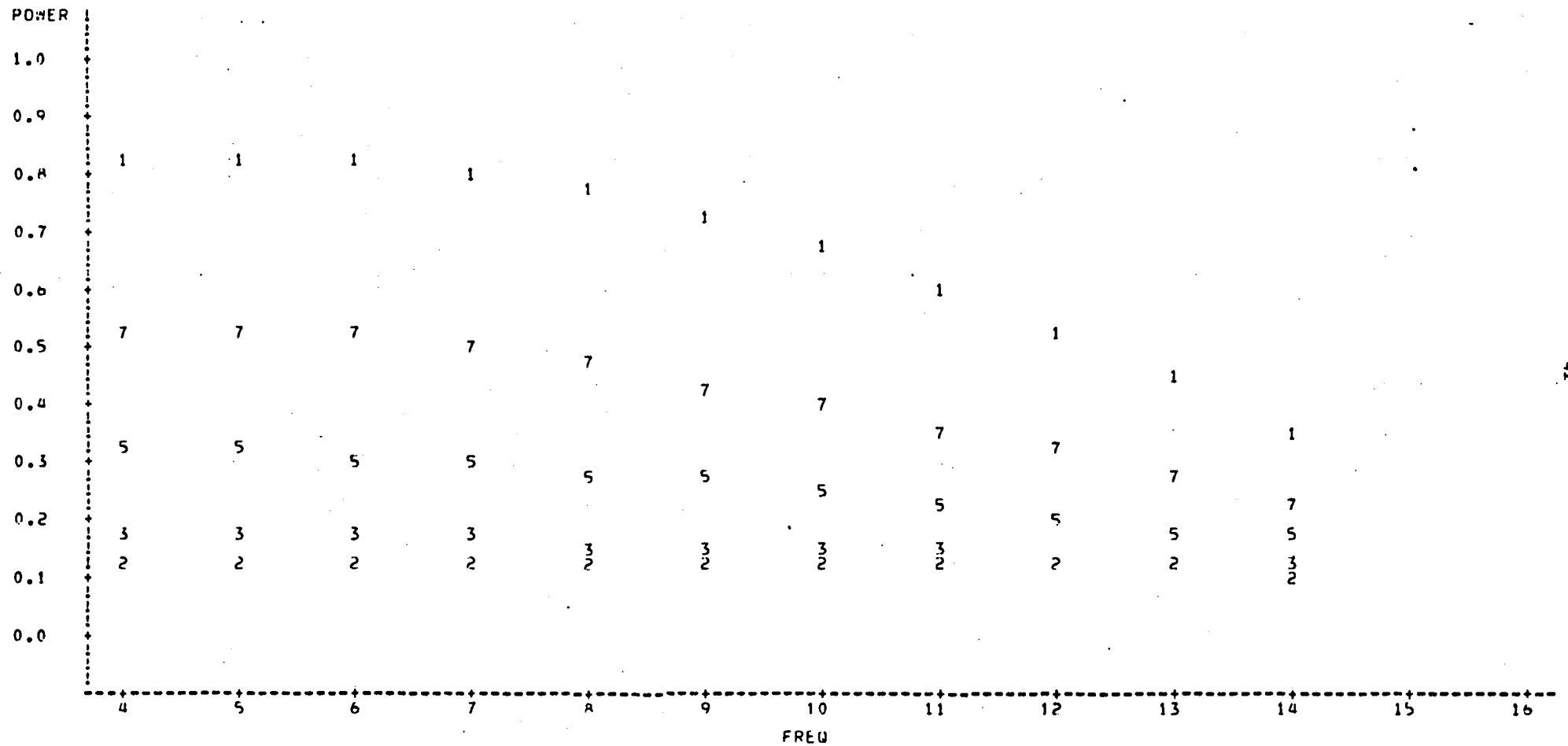


NOTE: 2 URS HIDDEN

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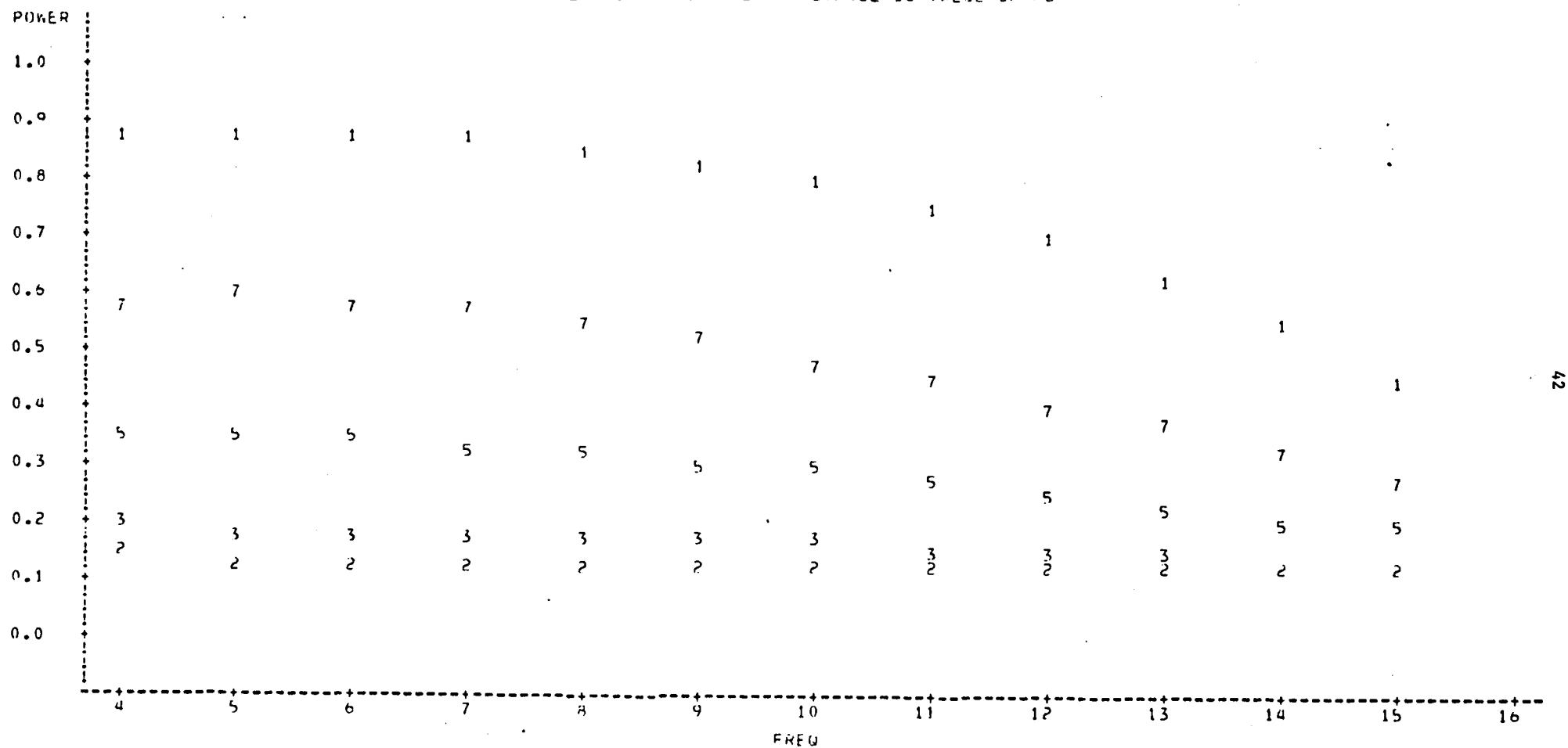


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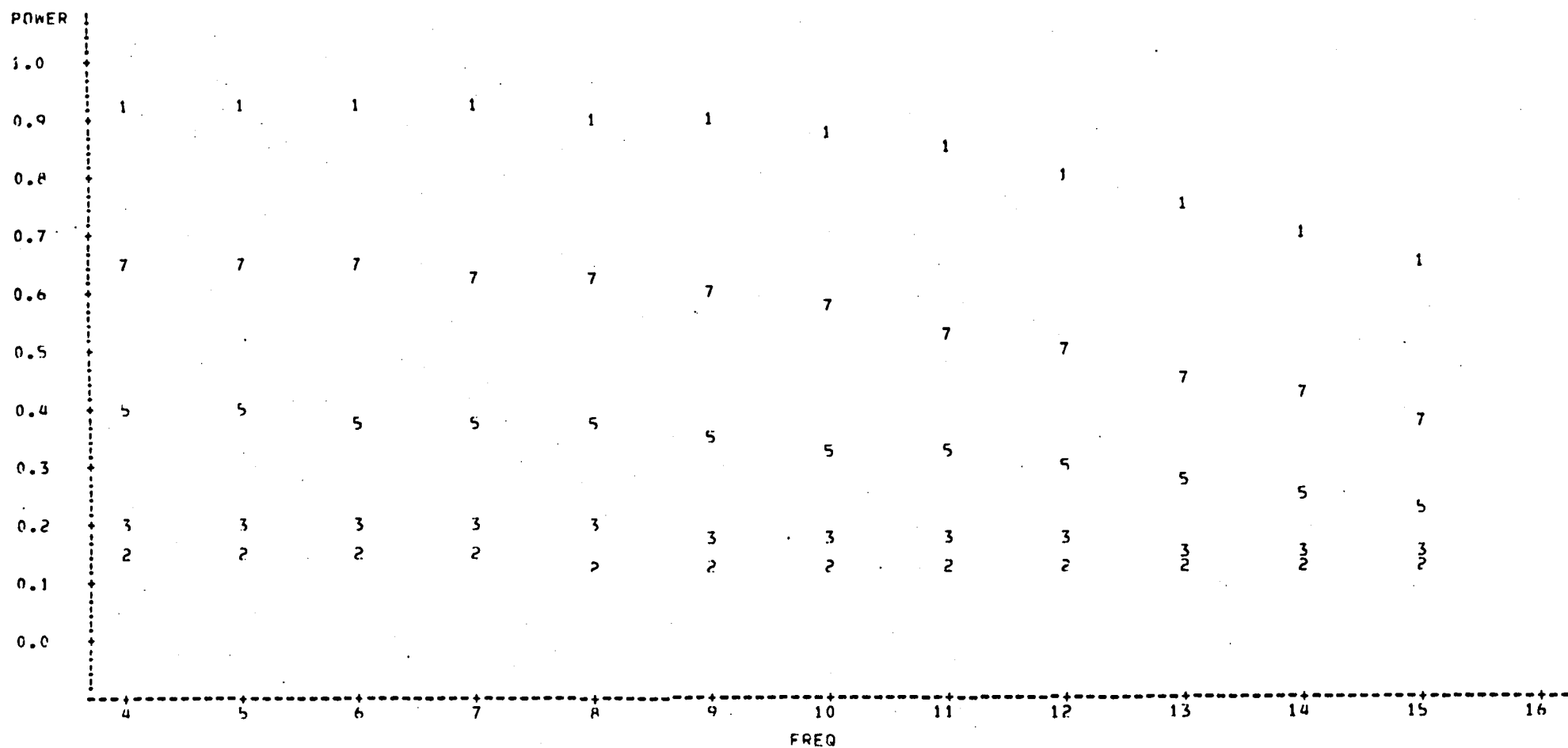
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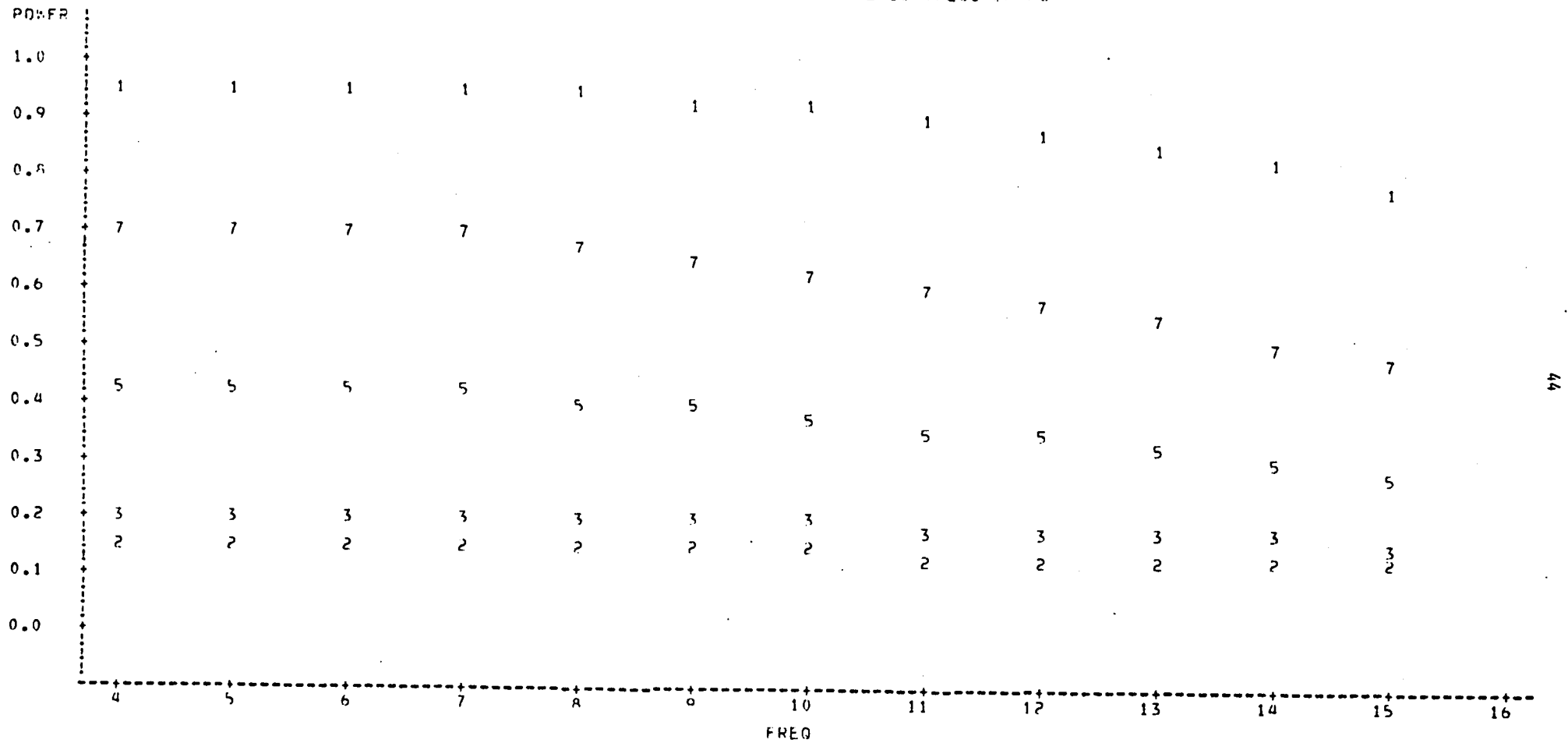
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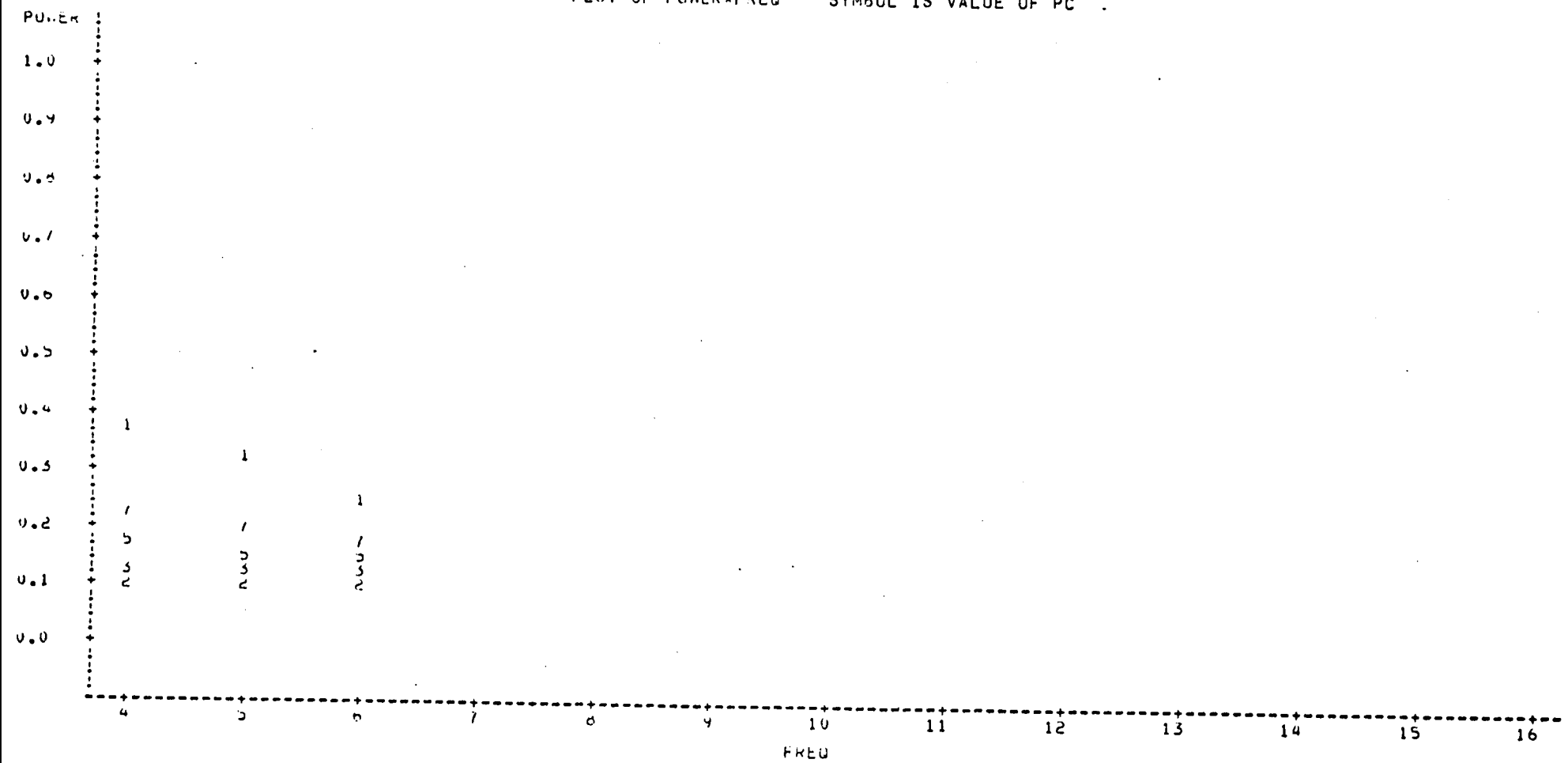


CORRELATION = .2

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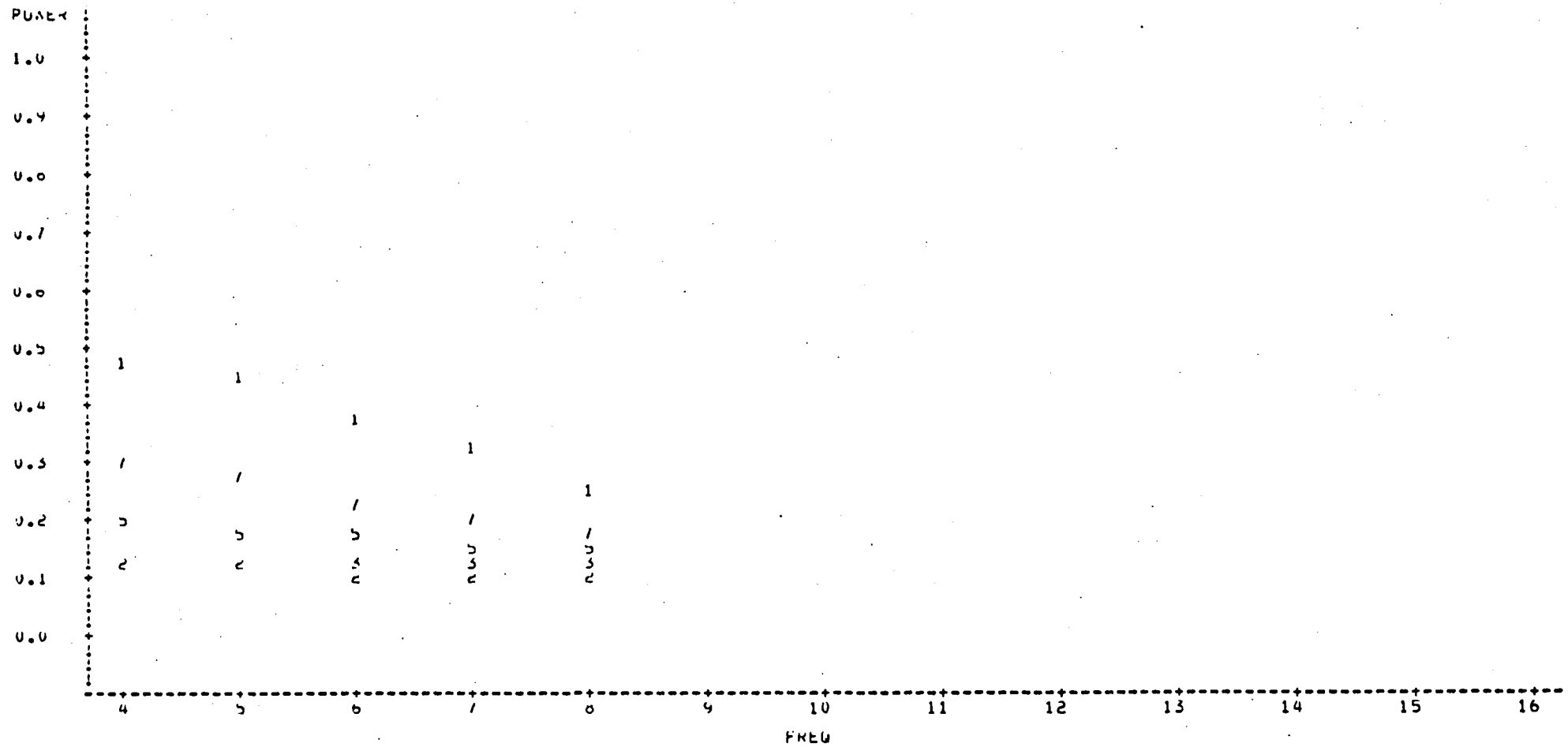
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STATISTICAL ANALYSIS SYSTEM
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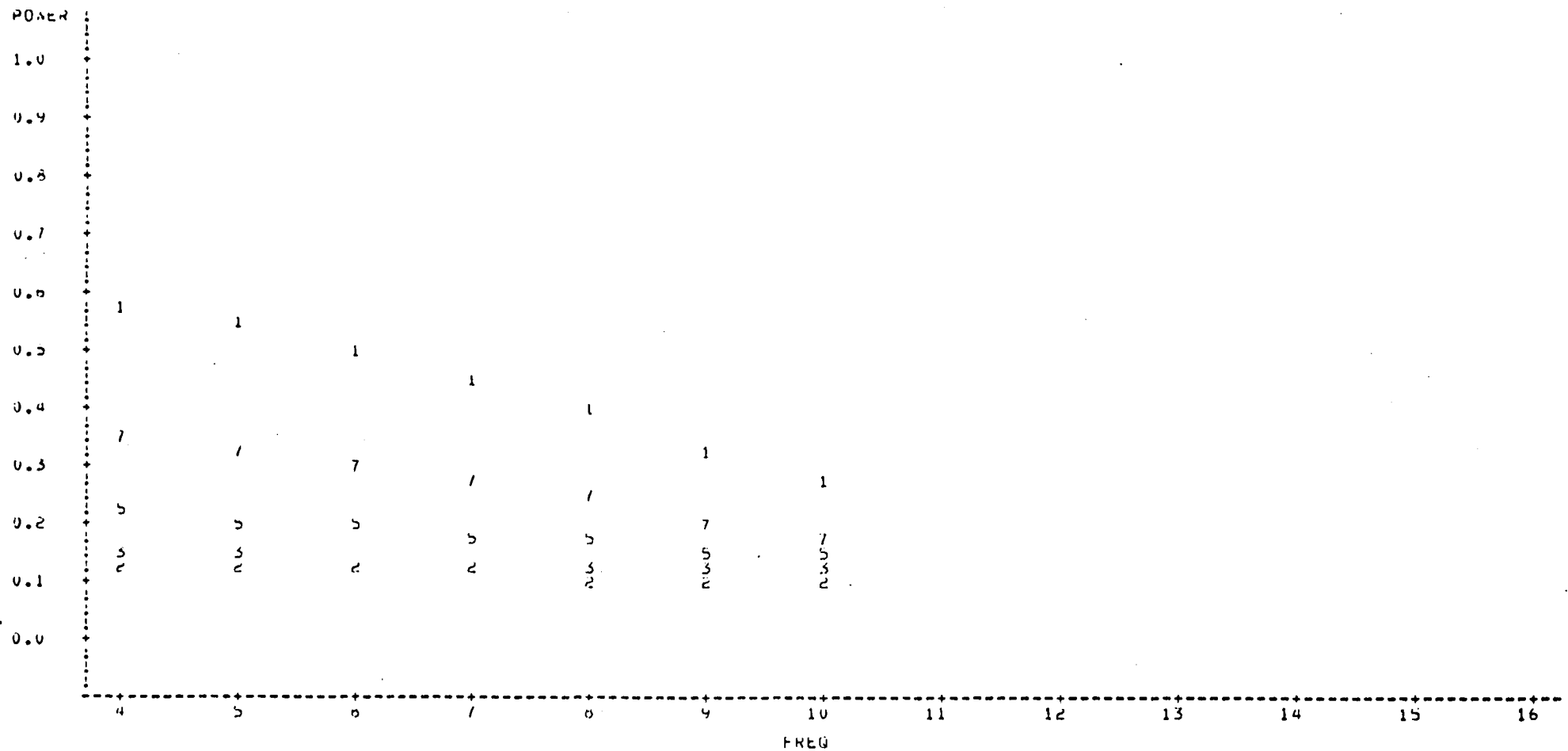


NOTE: 2 OBS HIDDEN

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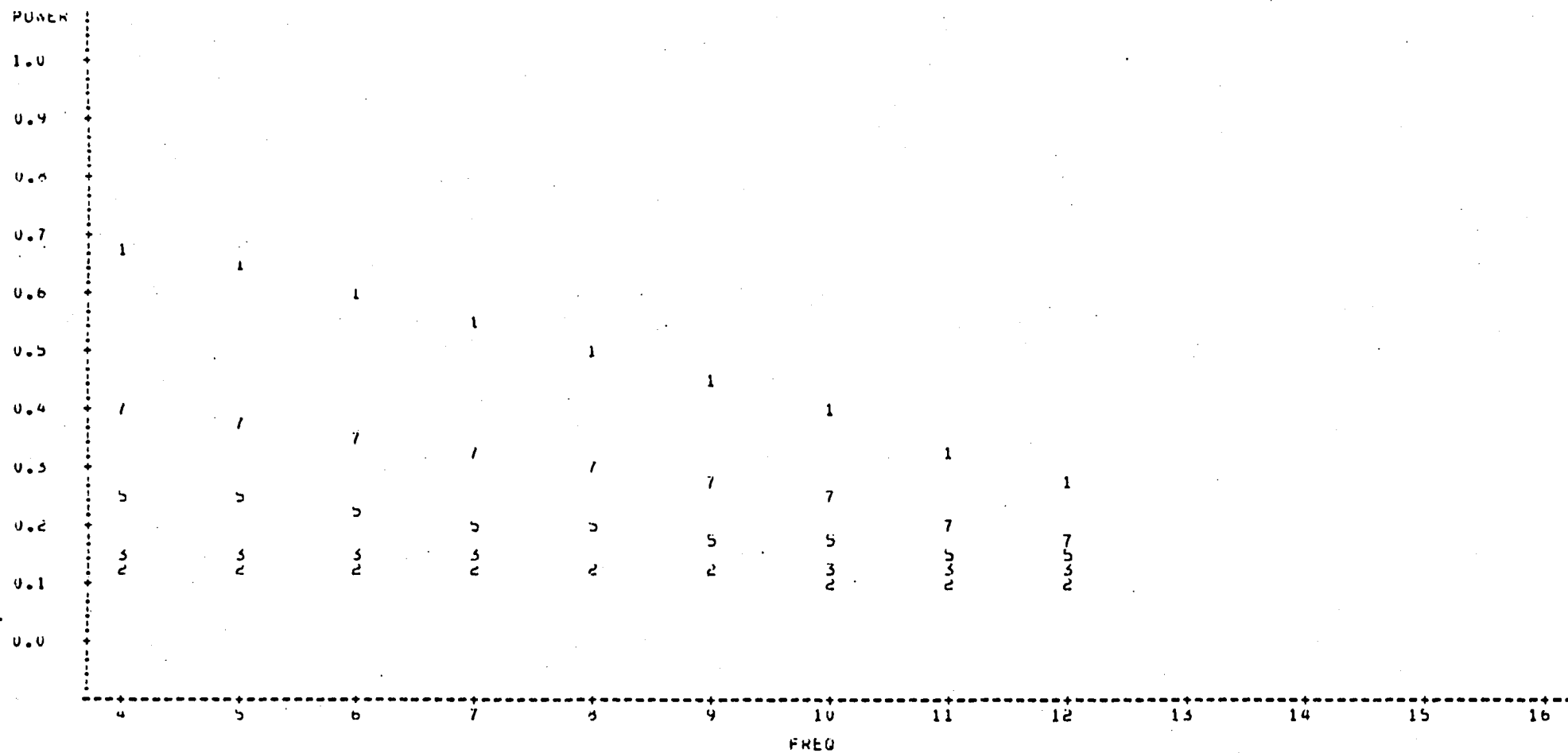


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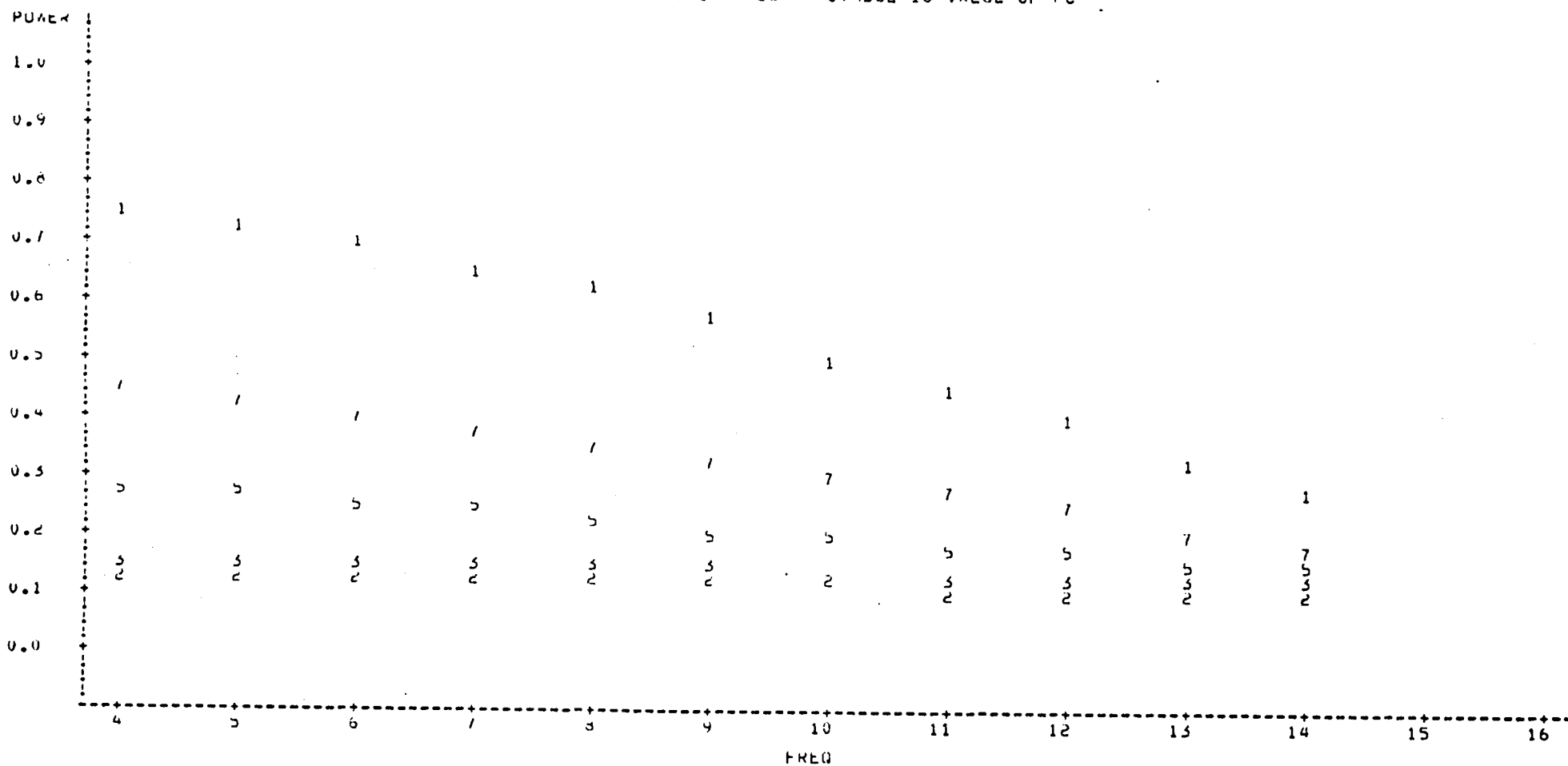


NOTE: 2 OBS HIDDEN

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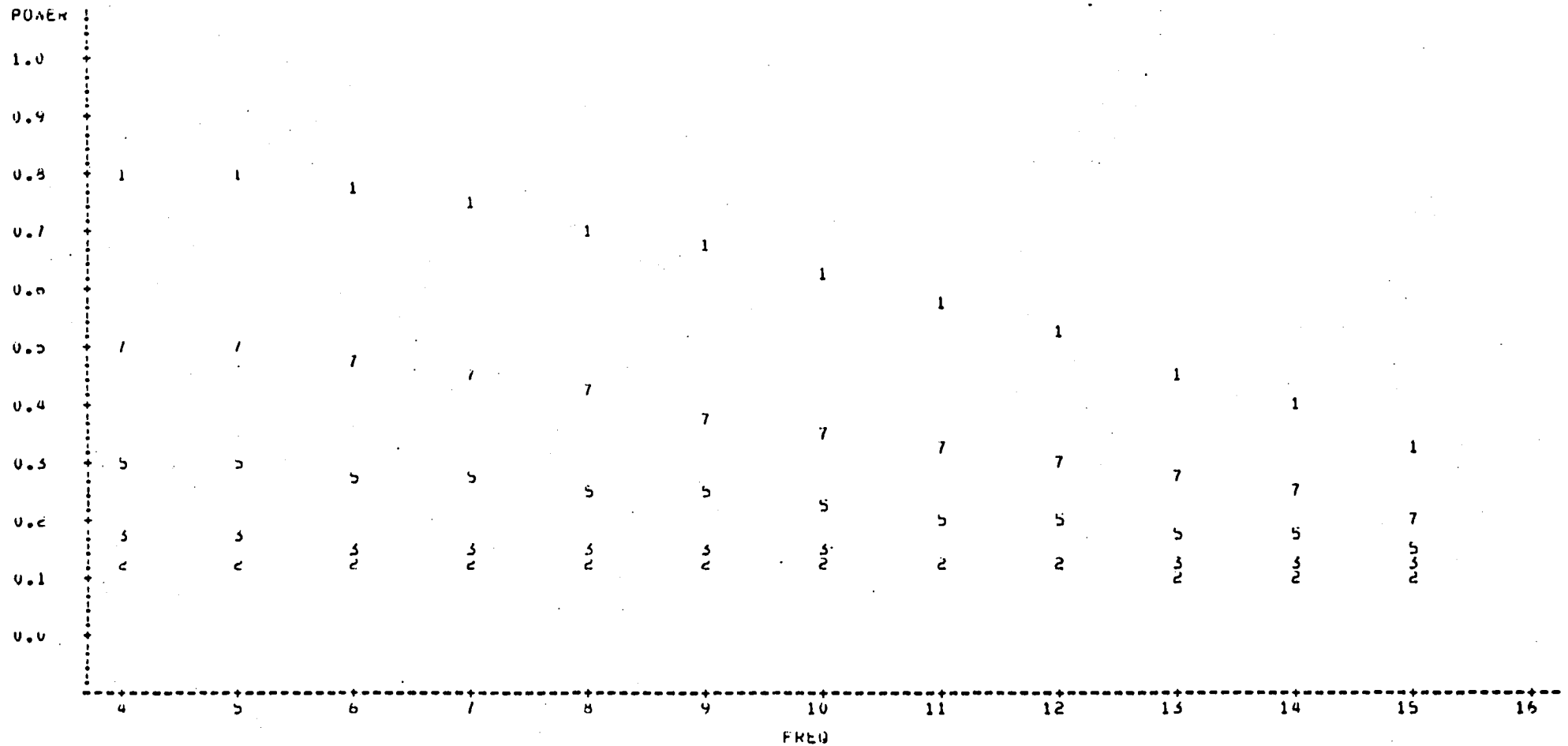


NOTE: 1 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM
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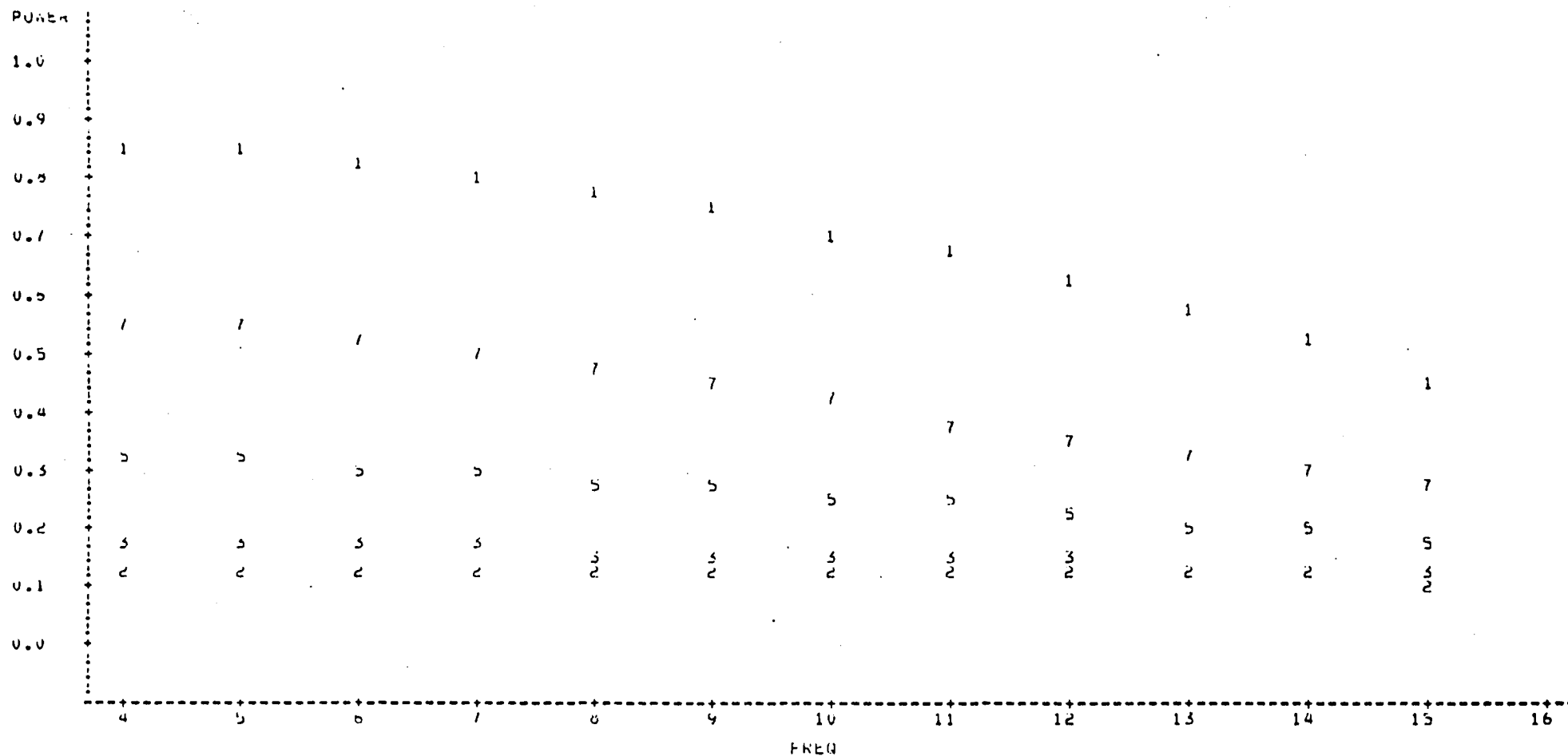


NOTE: 2 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM
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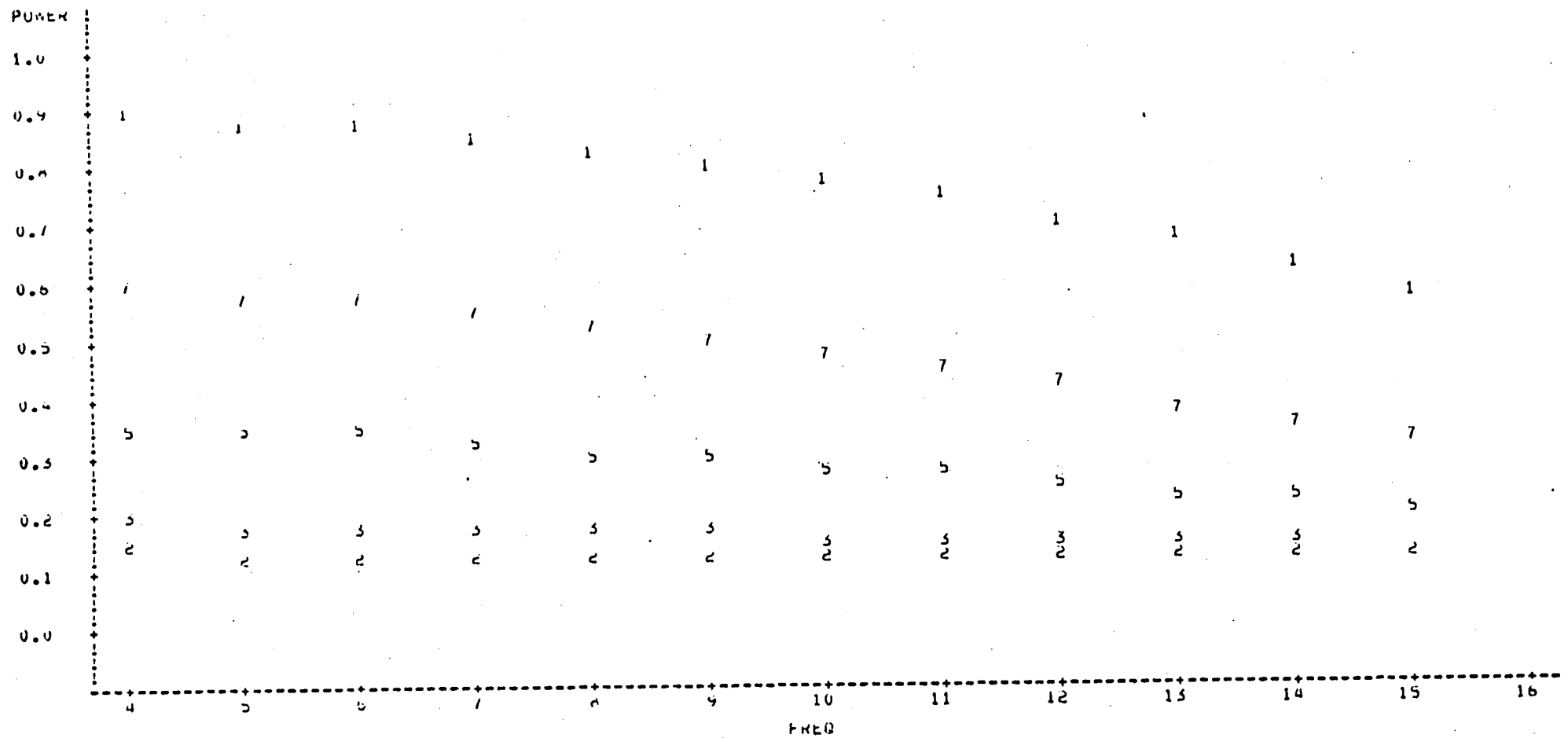


NOTE: 2 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=12

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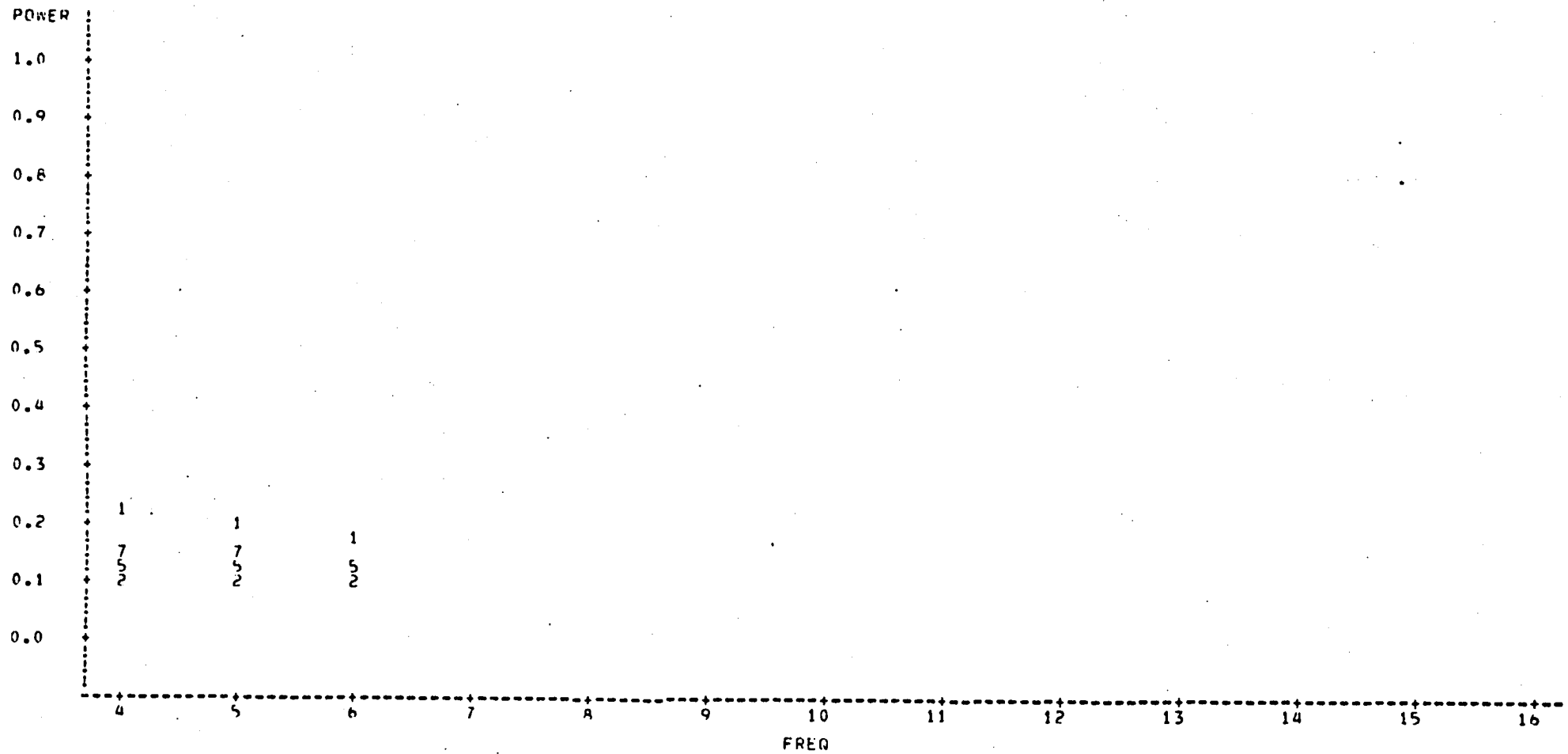
NOTE: 1 DMS FIDDER

HIGH CORRELATION

STATISTICAL ANALYSIS SYSTEM
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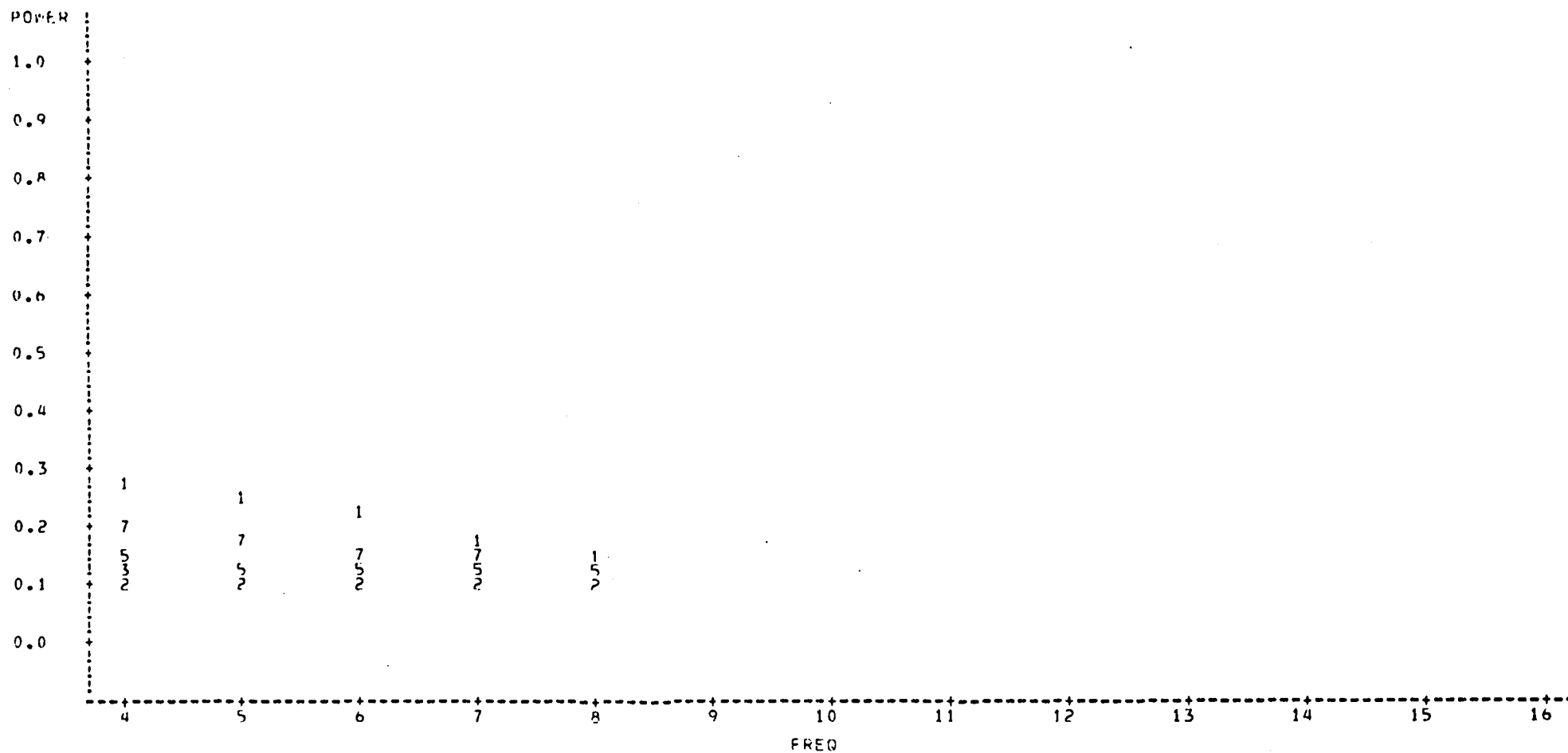


NOTE: 4 OBS HIDDEN

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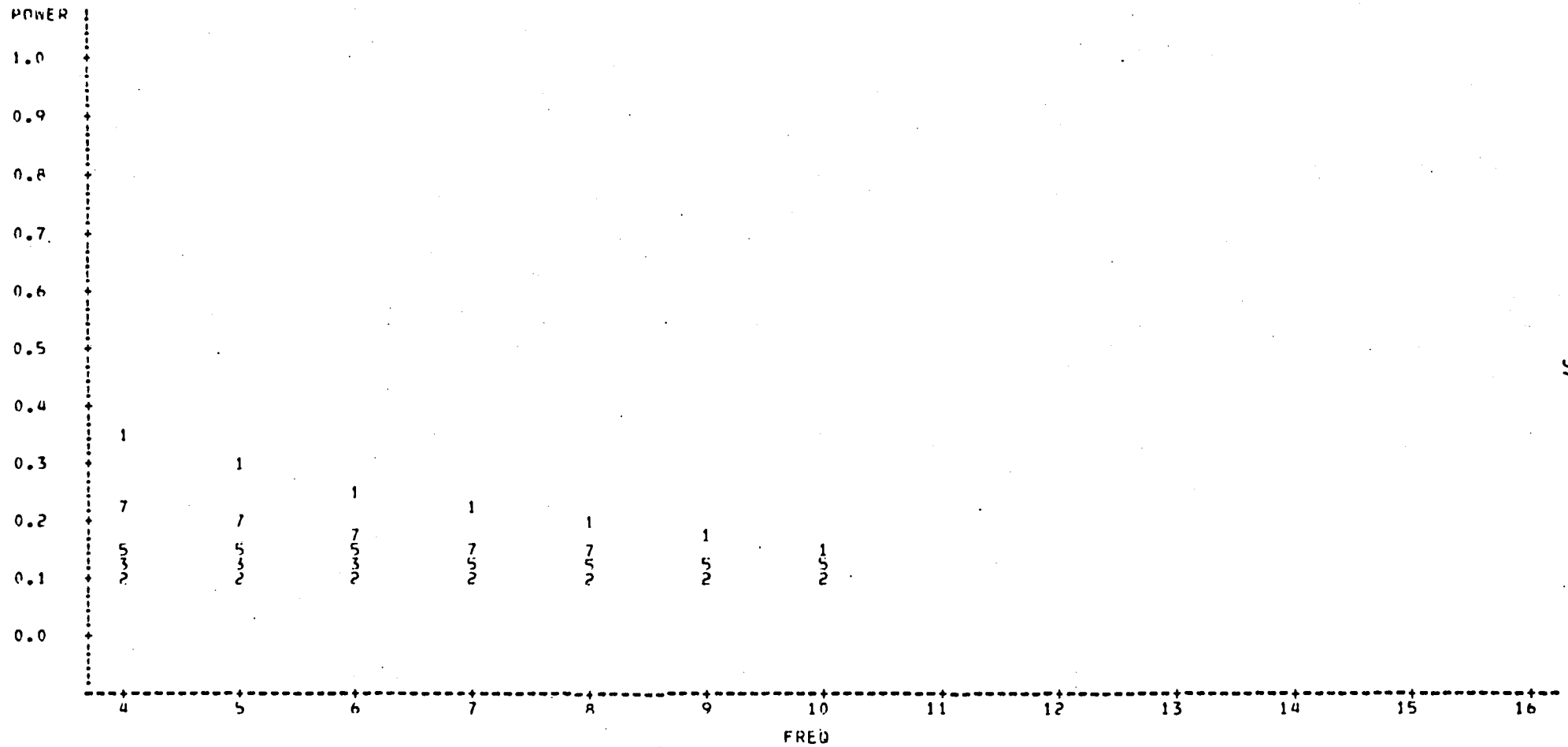


NOTE: 5 OBS HIDDEN

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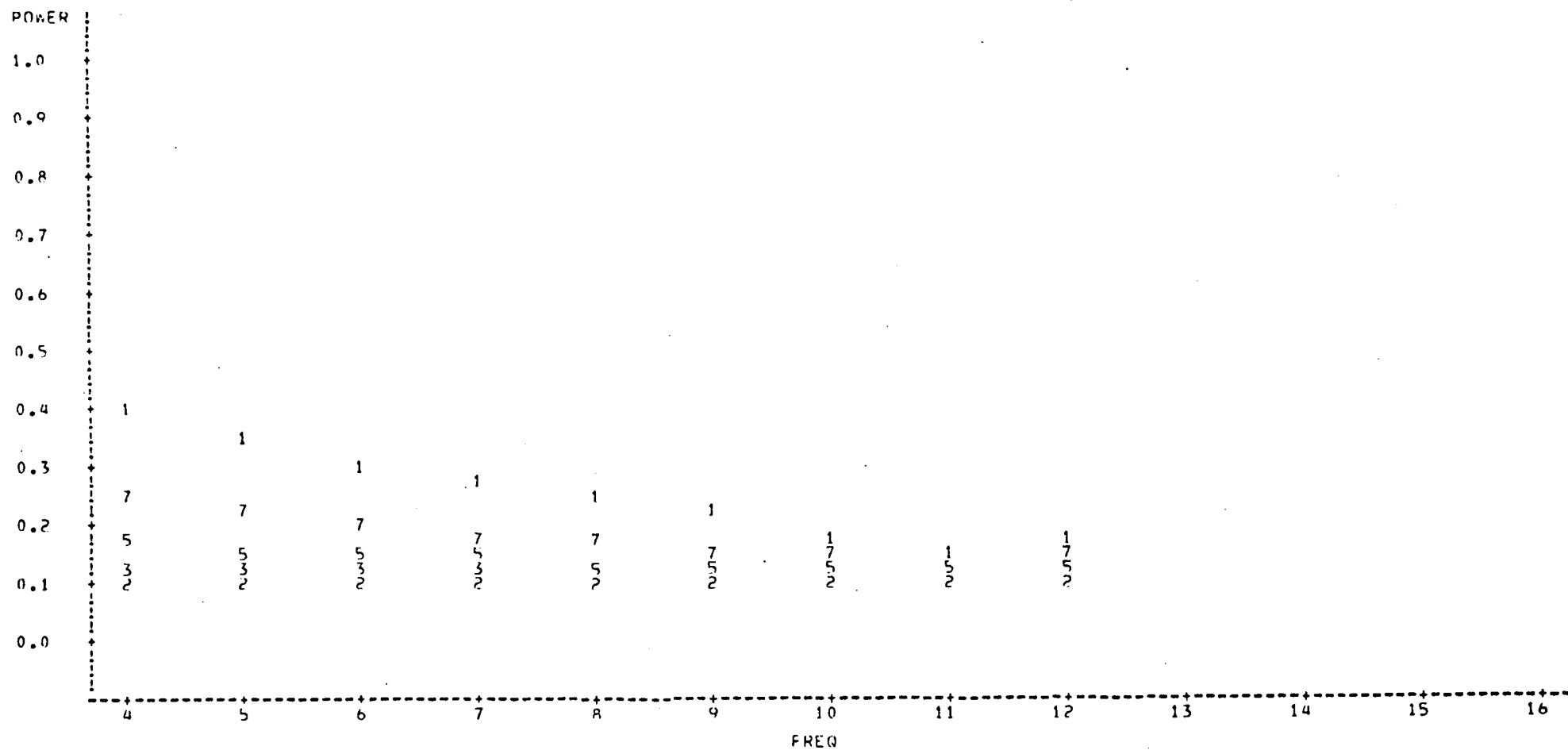


NOTE: 6 ORS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=8

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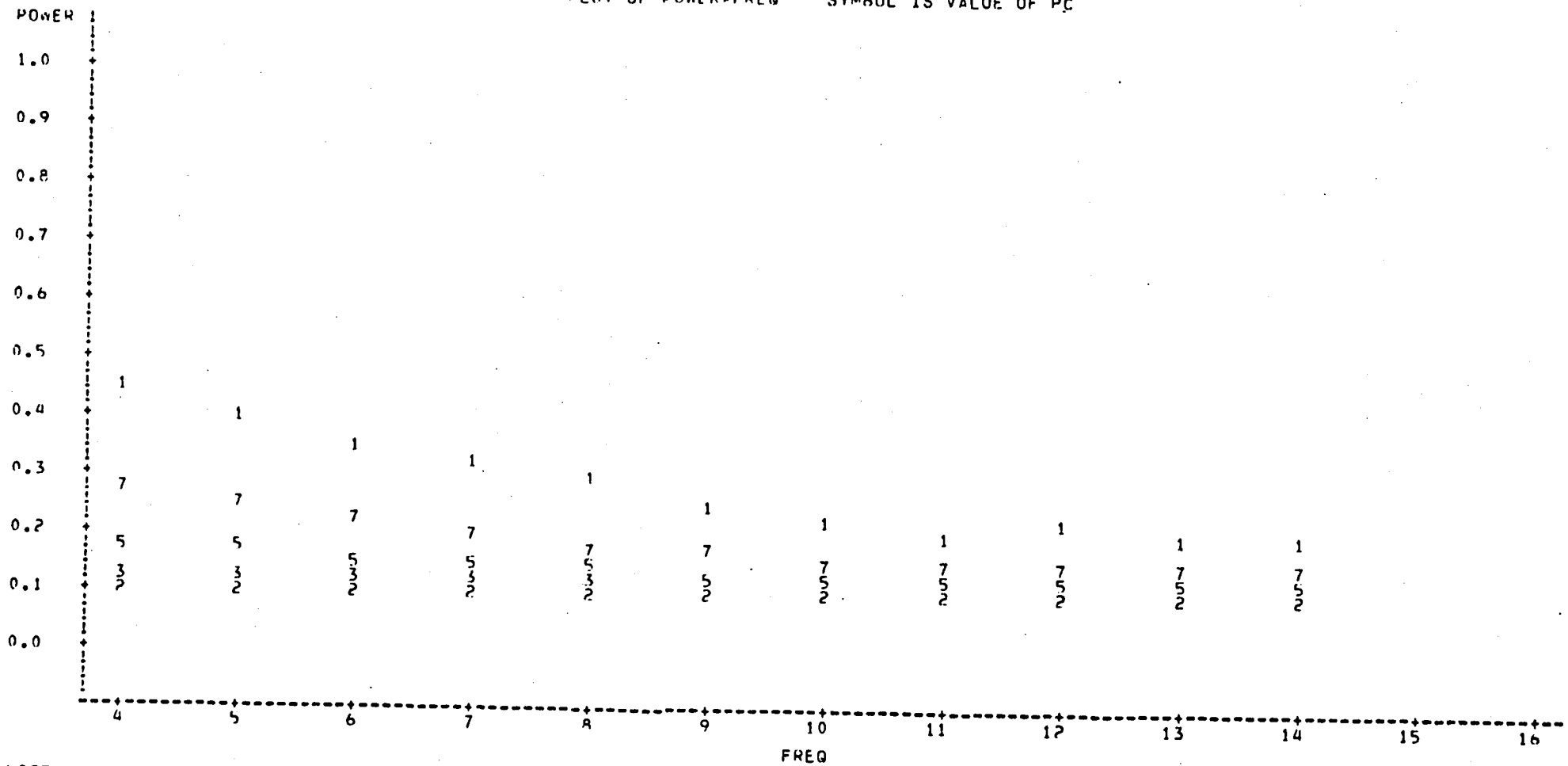


NOTE: 6 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=9

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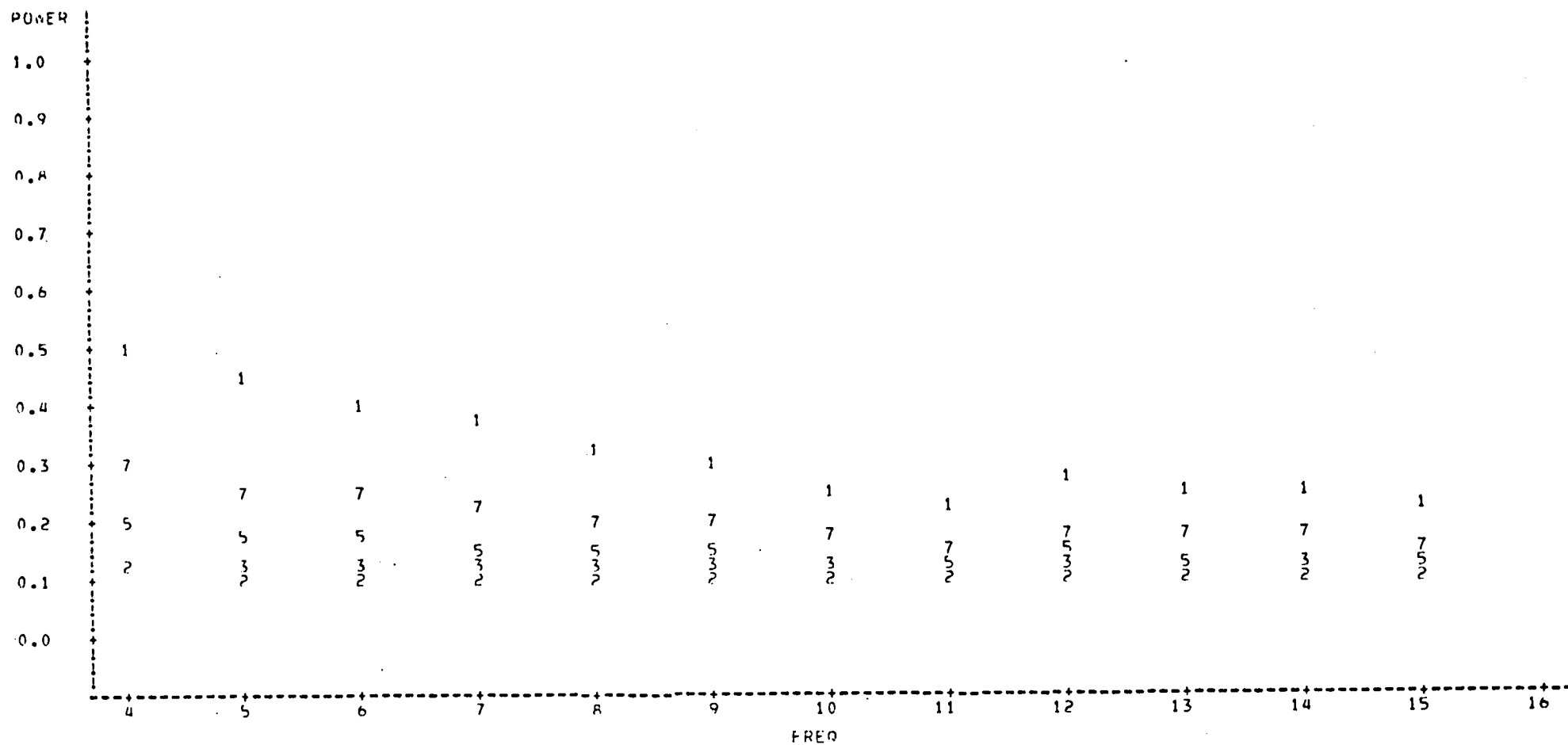


NOTE: 6 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=10

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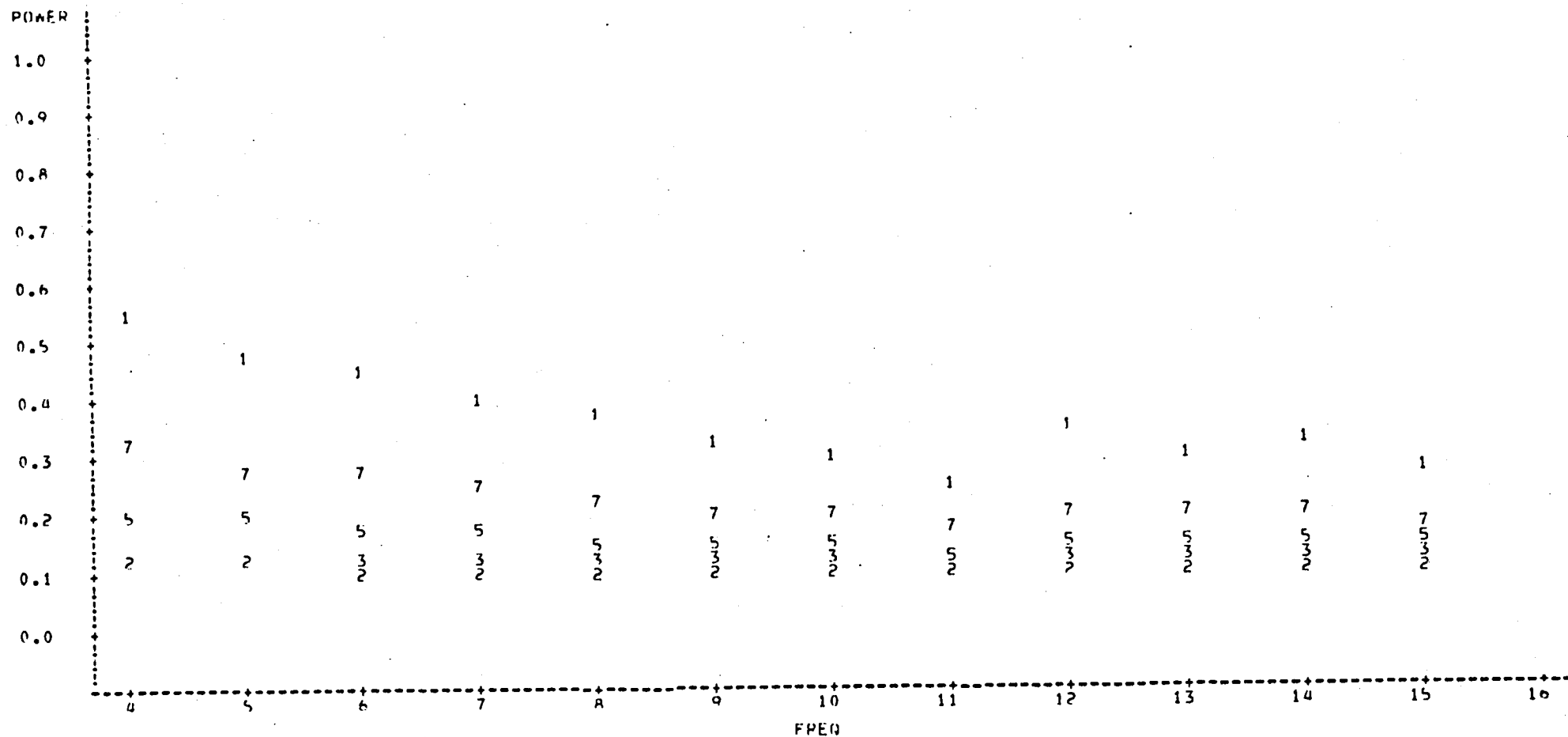


NOTE: 6 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM
N1=11

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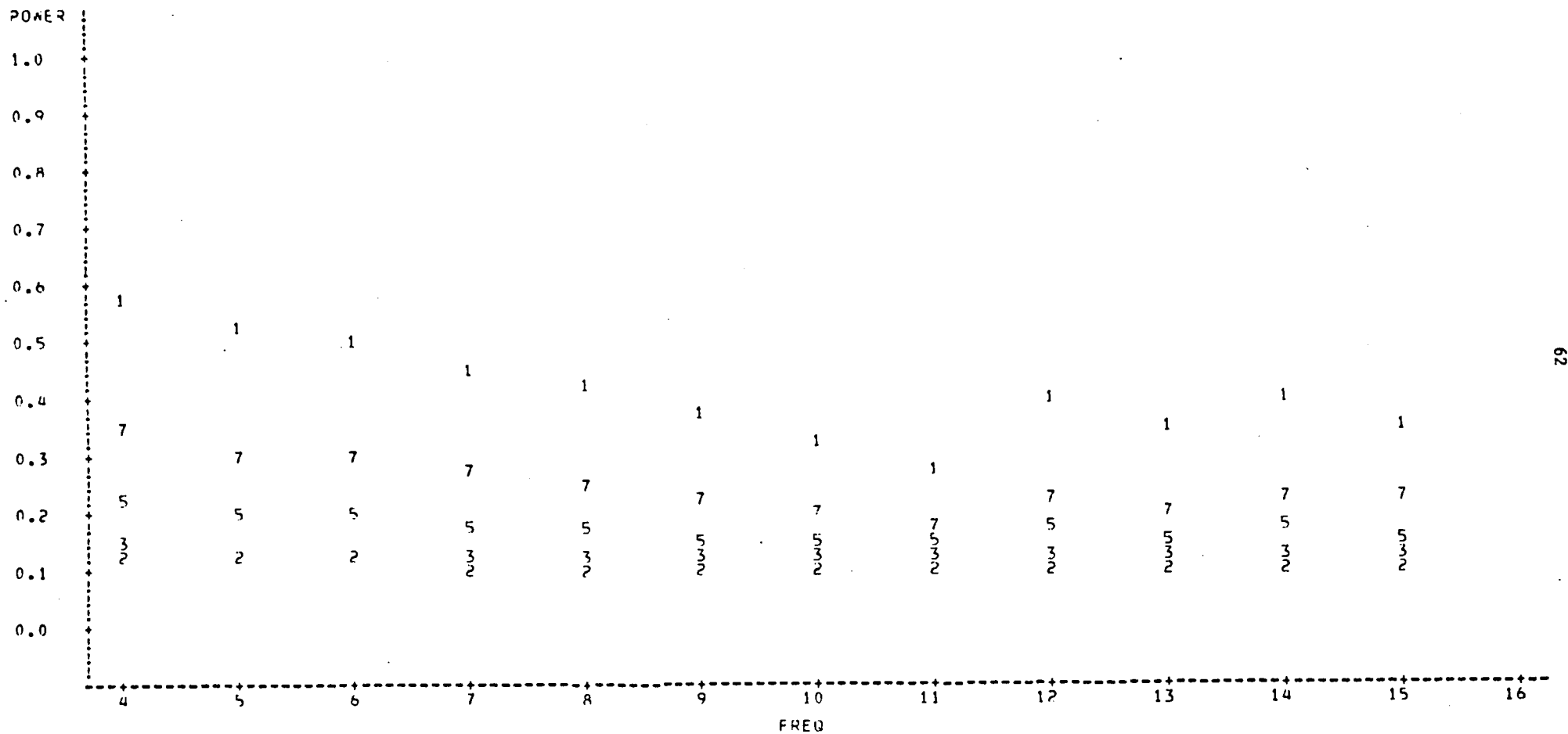


NOTE: 3 DMS HIDDEN

STATISTICAL ANALYSIS SYSTEM
N1=12

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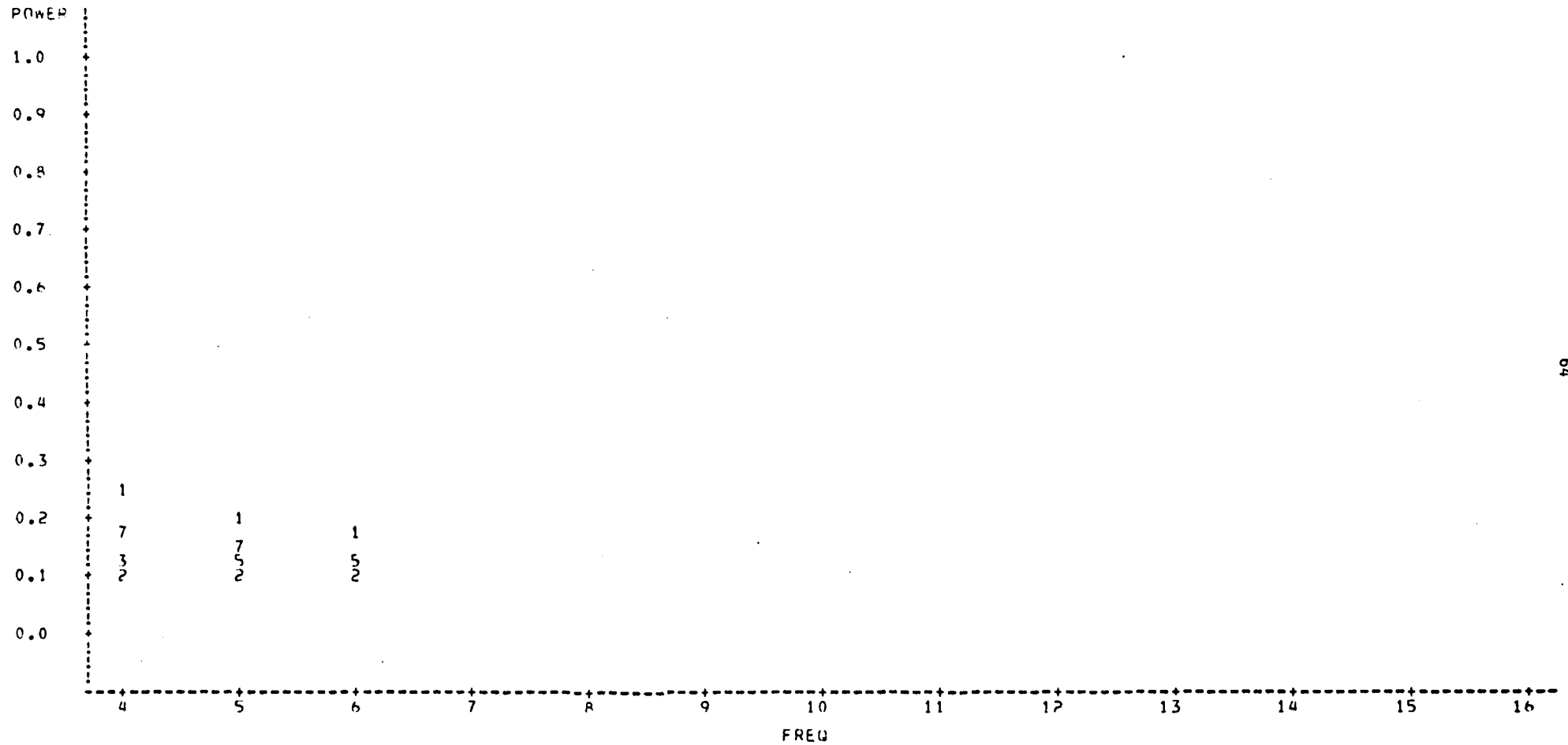
NOTE: 2 OBS HIDDEN

MIXED CORRELATION

STATISTICAL ANALYSIS SYSTEM N1=5

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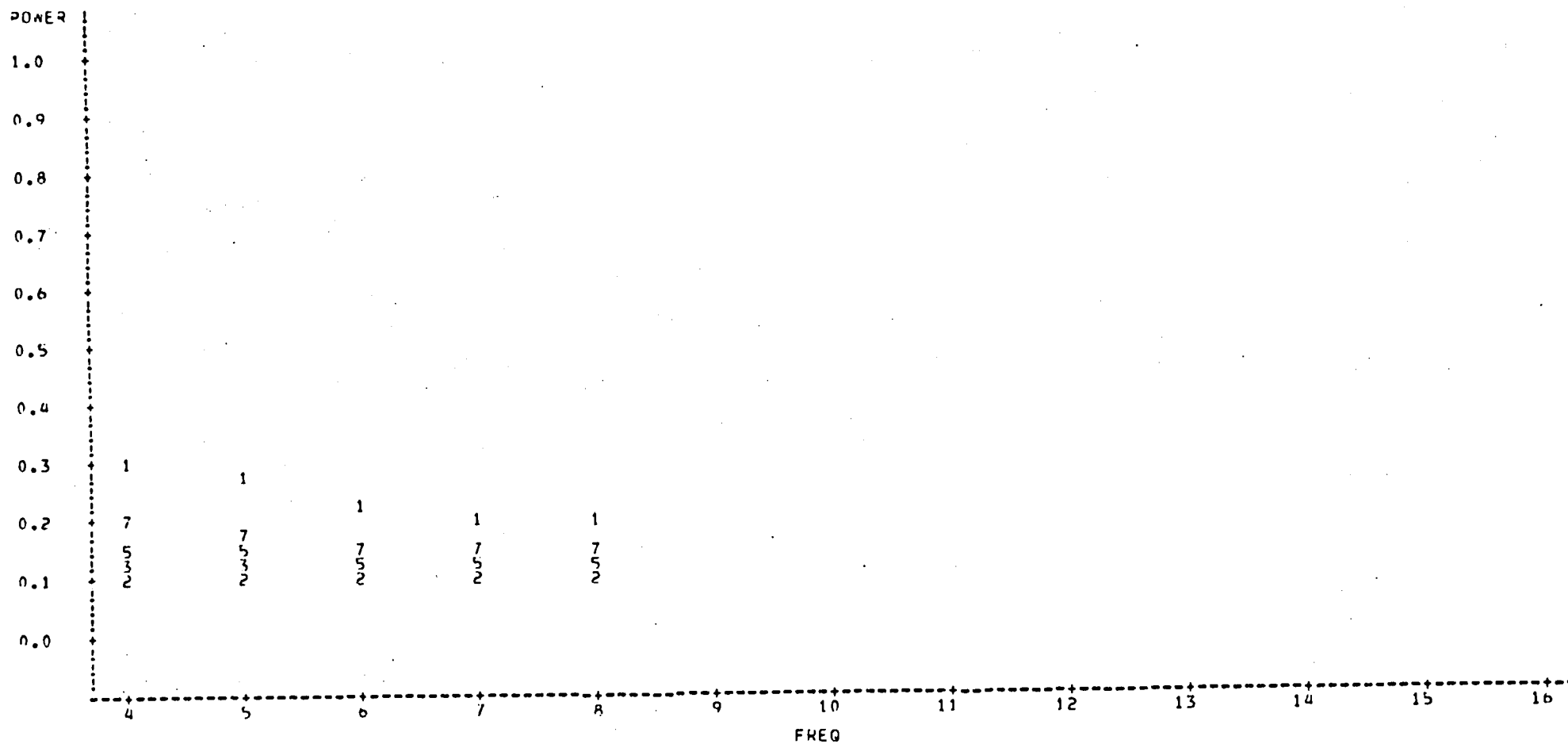


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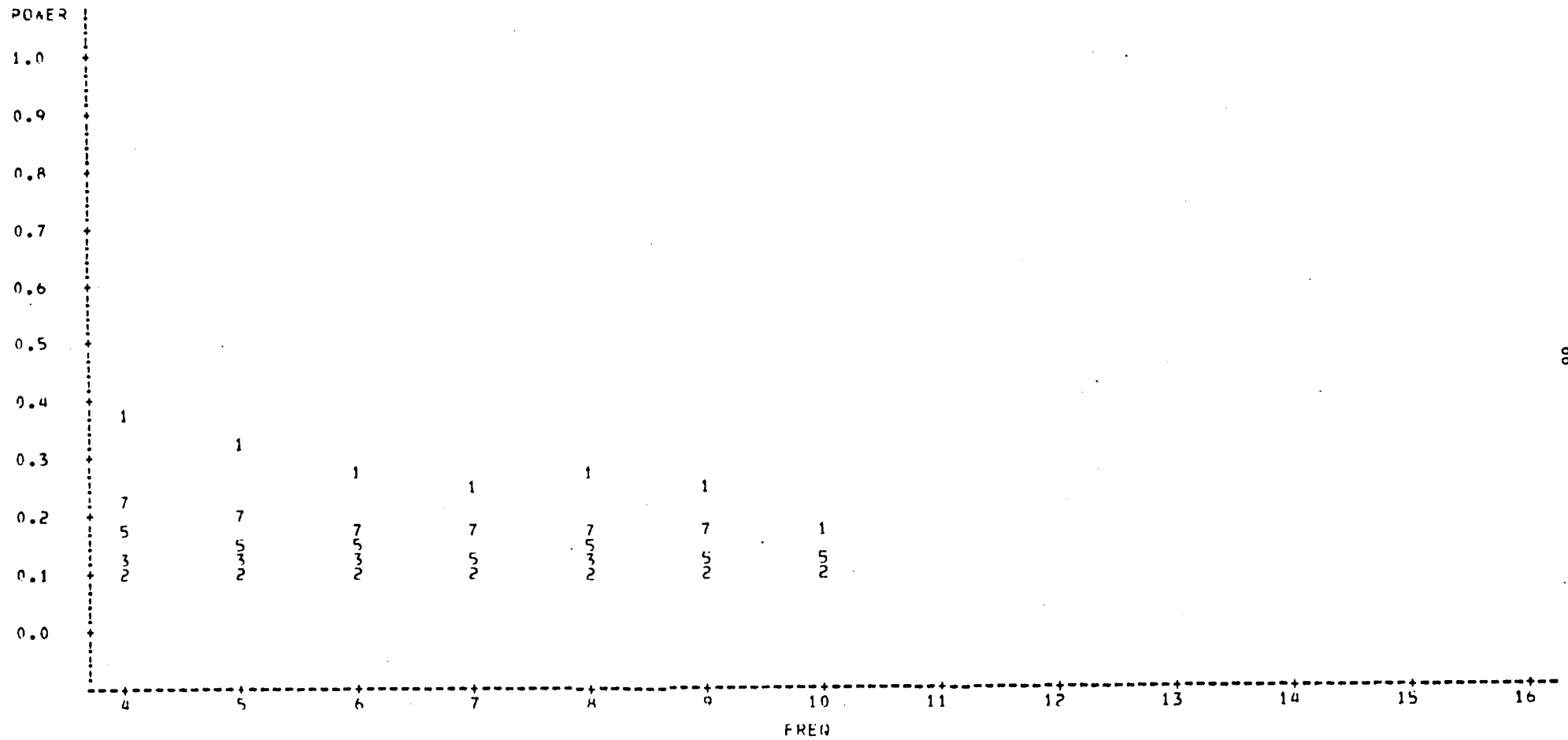


NOTE: 3 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=7

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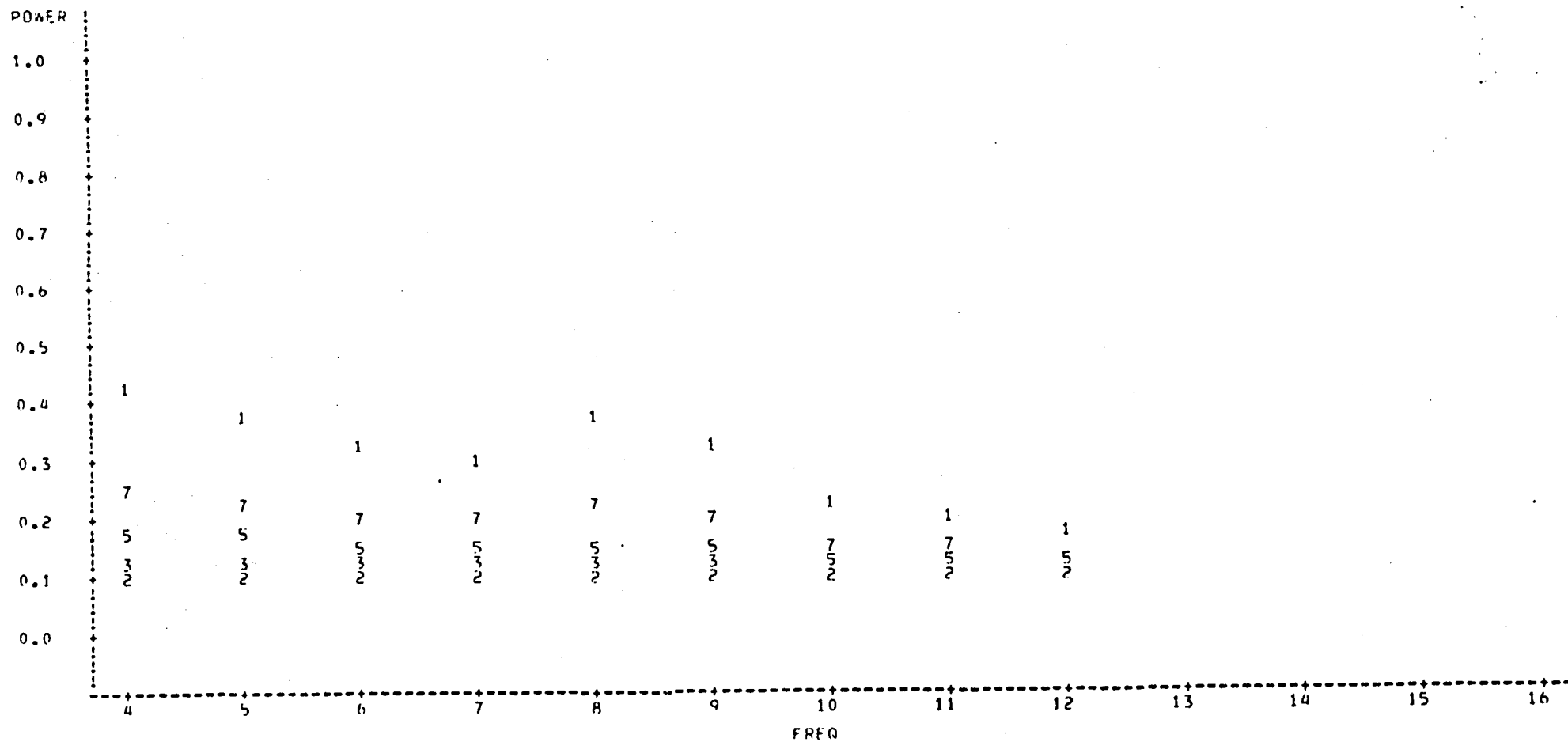


NOTE: 4 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=8

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PLOT OF POWER*FREQ SYMBOL IS VALUE OF PC



NOTE: 4 OBS HIDDEN

$N_1 = 9$

27

SYMBOL IS VALUE OF PC

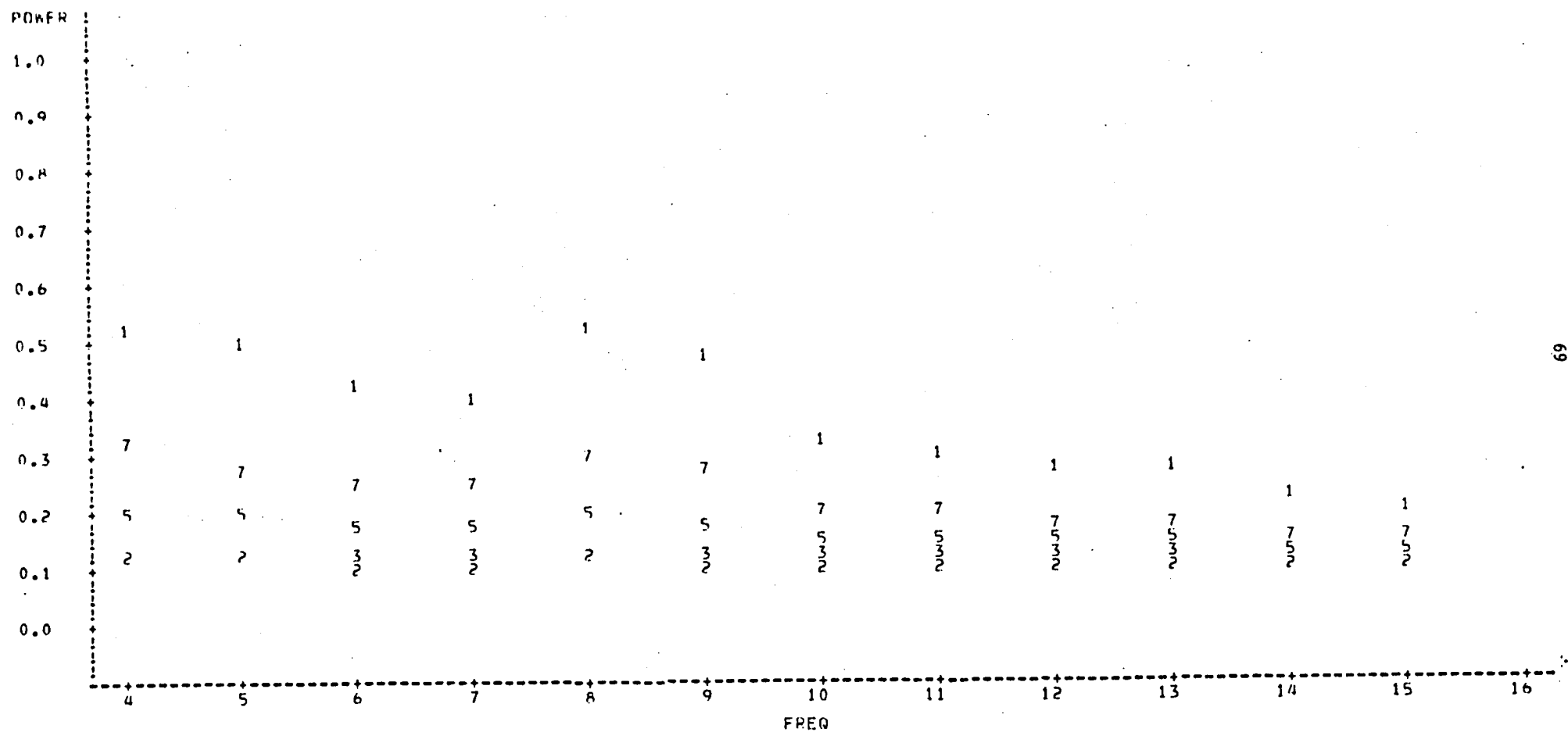


NOTE: 6 OBS HIDDEN

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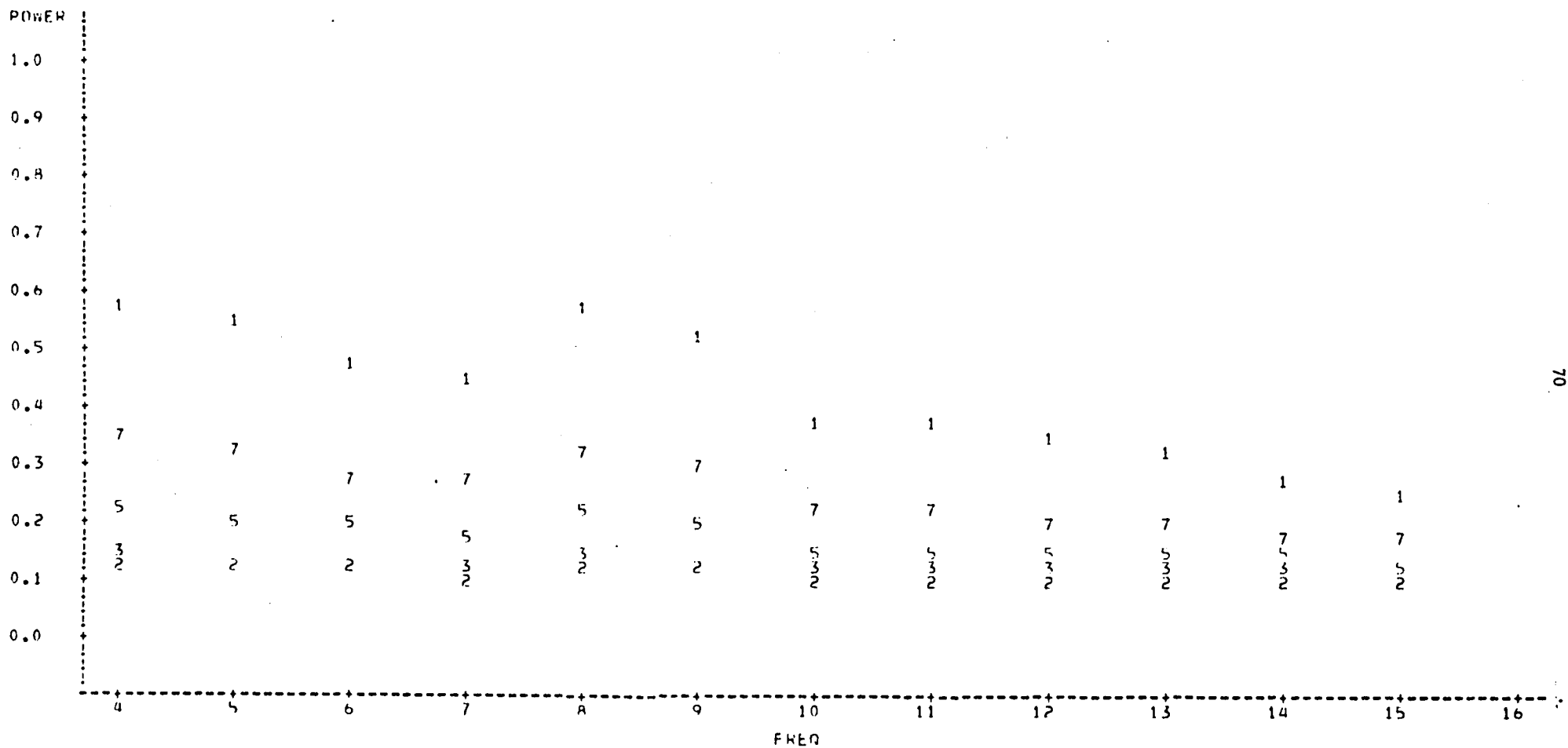


NOTE: 5 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM
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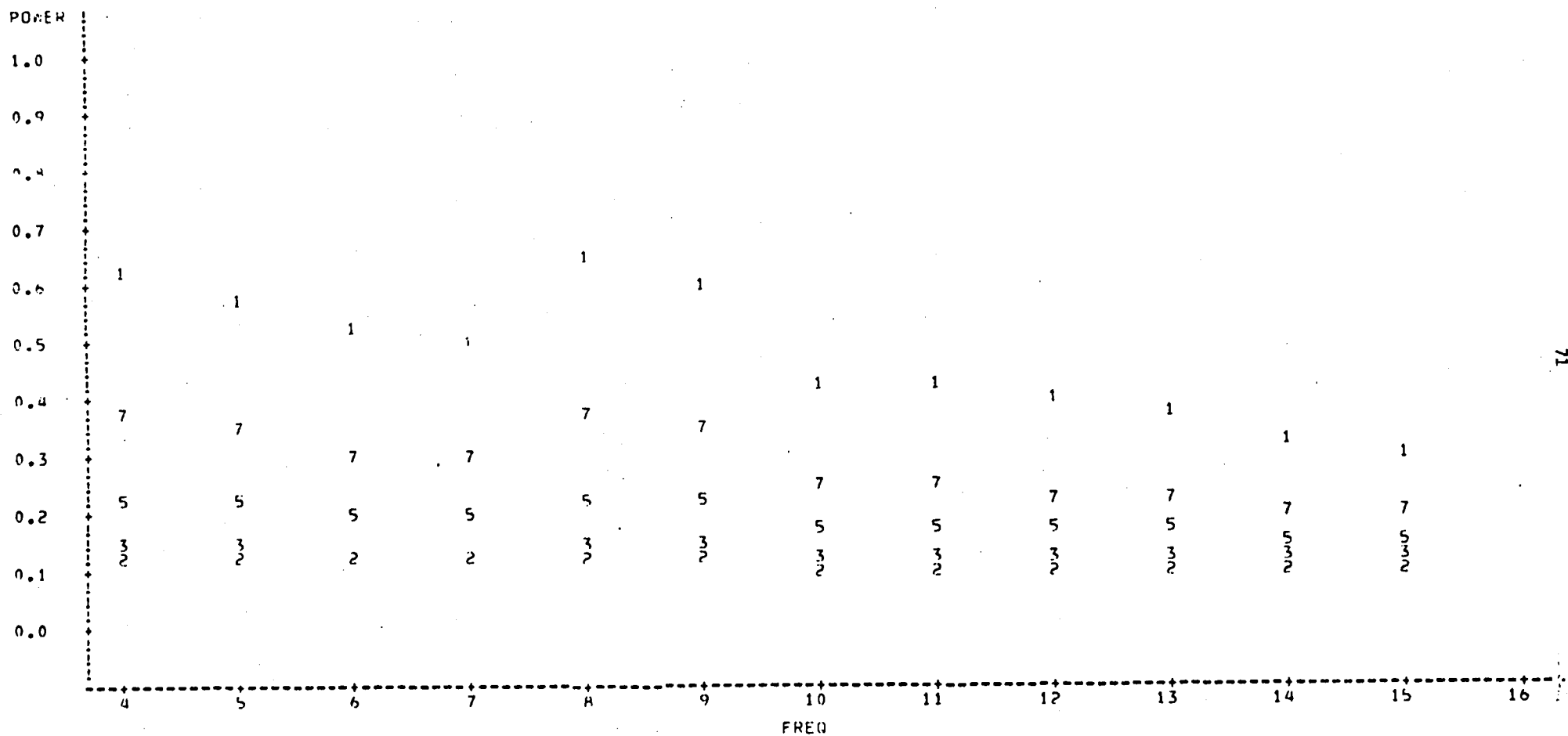


NOTE: 4 OBS HIDDEN

STATISTICAL ANALYSIS SYSTEM N1=12

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PLOT OF POWER*FREQ SYMBOL IS VALUE OF PC



NOTE: 2 OBS HIDDEN

Bibliography

1. Graybill, Franklin A., *Introduction to Linear Statistical Models*, Duxbury Press, 1976.
2. Morrison, Donald, *Multivariate Statistical Methods*, Second Edition McGraw-Hill, 1977.
3. Posten, Harry, "Power of the Likelihood-Ratio Test of the General Linear Hypothesis in Multivariate Analysis," Ph.D. dissertation, VPI&SU, 1960.

APPENDIX

Computer Program and Writeup

There are three subroutines supplied in the appendix for calculating various multivariate statistics for many hypothesis testing situations.

Some notation:

H - The hypothesis sum of squares matrix.

E - The error sum of squares matrix.

NP - The dimension of H and E.

NUH - The hypothesis degrees of freedom.

NUE - The error degrees of freedom.

The first subroutine ROY requires as input:

H, E - Square matrices of order NP stored columnwise in the vectors
H and E respectively.

NUH, NUE - Degrees of freedom are also supplied by the calling
program.

RTS - A work vector of length NP.

N - A work vector of length NP*NP.

Output:

RTS - Contains the ordered eigenvalues of HE^{-1} .

$\theta = \left(\frac{\lambda_{\max}}{1 + \lambda_{\max}} \right)$ where λ_{\max} is the largest eigenvalue of HE^{-1} .

LS, RM, RN are S, M, N respectively for use in Heck's charts.

The second subroutine NILKS needs as input H, E, NP, NUH, and NUE as previously described, where H and E are square NP*NP matrices stored columnwise in a vector of the same name.

Output:

$W1 = \frac{|E|}{|E+H|}$ for comparison with Wall's Tables.

$W2 = -(NUE - (NP - NUH + 1)/2) * LN(W1)$ for comparison with chi-square tables, in large samples where $W2$ has $NP * NUH$ degrees of freedom.

$W3 = F$ -ratio having $INUM$ numerator and $IDENOM$ denominator degrees of freedom when and exact F -test is available otherwise $W3=0$ and $INUM=IDENOM=0$.

The third subroutine HOTEL can calculate M T^2 -statistics. The necessary input includes

X - a vector of length $M * NP$ containing M sets of mean vectors of length NP stored columnwise.

S - Sigma - where $X(J)$, ($J=1, M$) has covariance-matrix $(1/A(J)) * \text{Sigma}$.

$A(J)$ - A vector of length n containing $\frac{n_1 n_2}{n_1 + n_2}$ (where the n_i are the number of replicates in the i th function), stored columnwise.

$T2$ - is a work vector of length M .

Output:

S : S -inverse

$T2$ - contains the M values of Hotelling's T^2 for direct comparison with tables

IEH - comes from matrix inversion routine

= 0 No error occurred during inversion

= -1 No result because of wrong input parameter n or the matrix is not positive definite

= k if there is a loss of significance.

SUBROUTINE ROY(H,E,W,NP,NUH,NUE,RTS,THETA,LS,RM,RN)

SUBROUTINES REQUIRED: DNRROOT AND DEIGEN SUPPLIED BY USER.

MATRICES H DUE TO HYPOTHESIS AND E DUE TO ERROR, DIMENSION NP, AND DEGREES OF FREEDOM, NUH FOR HYPOTHESIS AND NUE FOR ERROR, ARE SUPPLIED IN THE CALLING PROGRAM.

H AND E ARE SQUARE MATRICES OF ORDER NP STORED COLUMNWISE.

RTS AND X ARE WORK VECTORS OF DIMENSIONS NP AND NP*NP, RESPECTIVELY.

VALUES RETURNED ARE THE VECTOR RTS OF ORDERED LATENT ROOTS OF H*E-INVERSE, THETA=(LARGEST ROOT)/(1.+LARGEST ROOT), AND PARAMETERS LS, RM, AND RN FOR REFERENCE TO HECK'S CHARTS (DROP THE FIRST LETTER OF EACH).

DIMENSION H(1),E(1),W(1),RTS(1)

DOUBLE PRECISION H,E,RTS,THETA,X,W

IF (NUH-NP) 15,15,20

15 LS=NUH

GO TO 25

20 RS=NP

25 RM=(FLOAT(LABS(NUH-NP)-1))/2.

RN=(FLOAT(NUE-NP-1))/2.

CALL DNRROOT(EP,H,E,RTS,W)

THETA=RTS(1)/(1.00+RTS(1))

RETURN

END

SUBROUTINE WILKS(H,E,NP,NUH,NUE,W1,W2,W3,INUM,IDENOM,IEW)

SUBROUTINES REQUIRED: DMFSD FROM SSP LIBRARY.

MATRICES H DUE TO HYPOTHESIS AND E DUE TO ERROR, DIMENSION NP, AND DEGREES OF FREEDOM, NUH FOR HYPOTHESIS AND NUE FOR ERROR, ARE PROVIDED BY THE CALLING PROGRAM.

H AND E ARE STORED COLUMN-WISE AS UPPER TRIANGULAR MATRICES.

OUTPUT INCLUDES:

W1=DET(E)/DET(E+H) FOR COMPARISON WITH WALL'S TABLES,

W2=-(NUE-(NP-NUH+1))/2)*LN(W1) FOR COMPARISON WITH CHI-SQUARE HAVING

NP*NUH DEGREES OF FREEDOM IN LARGE SAMPLES (LN(.) IS NATURAL LOG).

W3=F-RATIO HAVING INUM NUMERATOR AND IDENOM DENOMINATOR DEGREES

OF FREEDOM WHEN EXACT F-TESTS ARE AVAILABLE; OTHERWISE W3=0.0,

INUM=0, AND IDENOM=0.

DIMENSION H(1),E(1)

DOUBLE PRECISION H,E,I,W1,W2,W3,WM,DETE,DETT,PROD

TOL=.000001

WM=(NP+1)/2

DO 20 I=1,L

I=4(I)+E(1)

20 H(1)=I

CALL DMFSD(L,NP,TOL,IEE)

```

CALL DMFSD(H,NP,TOL,IEH)
PRUD=1.00
L1=1
L2=2
DO 25 I=1,NP
  PRUD=(L1)*PRUD
  L1=L1+L2
25  L2=L2+1
  DETT=PRUD**2
  PRUD=1.00
  L1=1
  L2=2
  DO 30 I=1,NP
    PRUD=(L1)*PRUD
    L1=L1+L2
30  L2=L2+1
    DETT=PRUD**2
    A1=DETT/DETT
    AM=DFLOAT(NUE)-(DFLOAT(NP-NUH+1))/2.00
    A2=-AM*LOG(W1)
    IF(NUH.EQ.1) GO TO 50
    IF(NUH.EQ.2) GO TO 60
    IF(NP.EQ.1) GO TO 70
    IF(NP.EQ.2) GO TO 80
    GO TO 90
50  INUM=NP
    IDENOM=NUE+NUH-NP
    A3=(1.00-A1)*DFLOAT(IDENOM)/(W1*DFLOAT(INUM))
    GO TO 130
60  INUM=2*NP
    IDENOM=2*(NUE+NUH-NP-1)
    A3=(1.00-DSQRT(W1))*DFLOAT(IDENOM)/(DSQRT(W1)*DFLOAT(INUM))
    GO TO 130
70  INUM=NUH
    IDENOM=NUE
    A3=(1.00-W1)*DFLOAT(IDENOM)/(W1*DFLOAT(INUM))
    GO TO 130
80  INUM=2*NUH
    IDENOM=2*(NUE-1)
    A3=(1.00-DSQRT(W1))*DFLOAT(IDENOM)/(DSQRT(W1)*DFLOAT(INUM))
    GO TO 130
90  A3=0.00
    INUM=0
    IDENOM=0
130 IF(IEE.EQ.0) GO TO 135
    GO TO 145
135 IF(IEH.EQ.0) GO TO 140
    GO TO 145
140 IEA=0
    GO TO 150
145 IEA=1

```

150 RETURN
END

SUBROUTINE HOTEL(S,NP,X,A,T2,M,IEH)

SUBROUTINES REQUIRED: DSINV IN SSP LIBRARY

S IS A SYMMETRIC MATRIX OF ORDER NP STORED COLUMN-WISE IN UPPER

TRIANGULAR FORM.

A AND T2 ARE VECTORS OF LENGTH M.

X IS A VECTOR OF LENGTH NP.

ASSUME VECTORS X(J) TO HAVE COVARIANCE MATRICES $(1./A(J))*SIGMA$, FOR $J=1,M$. THE MATRIX S AND VECTORS X(J) ARE SUPPLIED BY THE CALLING PROGRAM AS WELL AS THE DIMENSION NP AND M. THE VECTORS X(J) ARE STORED COLUMN-WISE IN THE VECTOR X OF LENGTH NP*M.

ON RETURN S IS REPLACED BY SINVERSE, EACH X(J) IS REPLACED BY THE CORRESPONDING COEFFICIENTS OF A LINEAR DISCRIMINANT FUNCTION, AND THE VECTOR T2 CONTAINS VALUES OF THE HOTELLING'S T-SQUARE STATISTICS FOR DIRECT COMPARISON WITH SPECIAL AID TABLES (SEE JENSEN AND HOWE, 1967)

DIMENSION S(1),X(1),A(1),T2(1),M(1)

DOUBLE PRECISION S,X,A,T2,M,SUM,SUM1

TOL=.000001

CALL DSINV(S,NP,TOL,IEH)

L2=1

L3=NP

DO 50 I=1,M

SUM=0.00

L1=1

DO 25 J=L2,L3

DO 25 K=L2,J

SUM1=X(K)*S(L1)*X(J)

IF(J-K) 15,20,15

15 SUM1=SUM1+2.00

20 SUM=SUM+SUM1

25 L1=L1+1

T2(I)=SUM*A(I)

L2=L2+NP

30 L3=L3+NP

=0

L1=1

L2=NP

DO 50 I=1,M

L3=L2+NP

DO 40 J=L1,L2

SUM=0.00

DO 35 K=L1,L2

LJ=J-L3

LK=K-L3

CALL LOC(LJ,LK,JK,NP,NP,1)

SUM=SUM+S(JK)*X(K)

```

35 X(LJ)=SUM
40 CONTINUE
   DD 45 JJ=L1,L2
   JL=JJ-L3
45 X(JJ)= (JL)
   L=L+1
   L1=L1+NP
50 L2=L2+NP
   RETURN
END

```

```

      SUBROUTINE DNRROOT(M,A,B,XL,X)
      DIMENSION A(1),B(1),XL(1),X(1)
      DOUBLE PRECISION A,B,XL,X,SUMV
      K=1
      DD 100 J=2,M
      L=M*(J-1)
      DD 100 I=1,J
      L=L+1
      K=X+1
100 X(K)=B(L)
      K=0
      CALL OEIGEN(B,X,M,MV)
      L=0
      DD 110 J=1,M
      L=L+J
110 XL(J)=1./SUMV-1/(OAPS(X(L)))
      K=0
      DD 115 J=1,M
      DD 115 I=1,M
      K=X+1
115 B(K)=X(K)*XL(J)
      DD 120 I=1,M
      VP=0
      DD 120 J=1,M
      V1=M*(I-1)
      L=M*(J-1)+I
      X(L)=0.D0
      DD 120 K=1,M
      V1=V1+1
      VP=VP+1
120 X(L)=X(L)+B(V1)*A(VP)
      L=L+1
      DD 130 J=1,M
      DD 130 I=1,J
      V1=1-M
      VP=M*(J-1)
      L=L+1
      A(L)=0.D0
      DD 130 K=1,M
      V1=V1+M

```

```

NR00 370
NR00 380
NR00 460
NR00 610
NR00 620
NR00 630
NR00 640
NR00 650
NR00 660
NR00 670
NR00 710
NR00 720
NR00 770
NR00 780
NR00 790
NR00 800
NR00 810
NR00 820
NR00 830
NR00 840
NR00 850
NR00 890
NR00 900
NR00 910
NR00 920
NR00 930
NR00 940
NR00 950
NR00 960
NR00 970
NR00 980
NR00 990
NR001000
NR001010
NR001020
NR001030
NR001040
NR001050
NR001060
NR001070

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DATE 03/31/78

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```

      NP=NP+1
130  A(L)=A(L)+X(N1)*H(N2)
      CALL DEIGEN(A,X,M,VV)
      L=L+1
      DO 140 I=1,M
      L=L+1
140  XL(I)=A(L)
      DO 150 I=1,M
      V2=0
      DO 150 J=1,M
      V1=1-M
      L=*(J-1)+1
      A(L)=0.00
      DO 150 I=1,M
      V1=1+M
      V2=NP+1
150  A(L)=A(L)+H(N1)*X(N2)
      L=L+1
      DO 160 J=1,M
      SUMV=0.00
      DO 170 I=1,M
      L=L+1
170  SUMV=SUMV+A(L)*A(L)
175  SUMV=SQRT(SUMV)
      DO 180 I=1,M
      X=X+1
180  X(K)=A(K)/SUMV
      SUMV=0.00

```

```

NR001080
NR001090
NR001130
NR001140
NR001150
NR001160
NR001170
NR001210
NR001220
NR001230
NR001240
NR001250
NR001260
NR001270
NR001280
NR001290
NR001300
NR001310
NR001320
NR001340
NR001340
NR001350
NR001360
NR001370
NR001380
NR001390
NR001400
NR001410
NR001420
NR001430

```

SUBROUTINE EIGEN

PURPOSE

COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC
MATRIX

USAGE

CALL EIGEN(A,R,N,MV)

DESCRIPTION OF PARAMETERS

A - ORIGINAL MATRIX (SYMMETRIC), DESTROYED IN COMPUTATION.
RESULTANT EIGENVALUES ARE DEVELOPED IN DIAGONAL OF
MATRIX A IN DESCENDING ORDER.

R - RESULTANT MATRIX OF EIGENVECTORS (STORED COLUMNWISE,
IN SAME SEQUENCE AS EIGENVALUES)

N - ORDER OF MATRICES A AND R

MV- INPUT CODE

0 COMPUTE EIGENVALUES AND EIGENVECTORS

1 COMPUTE EIGENVALUES ONLY (R NEED NOT BE
DIMENSIONED BUT MUST STILL APPEAR IN CALLING
SEQUENCE)

REMARKS

ORIGINAL MATRIX A MUST BE REAL SYMMETRIC (STORAGE MODE=1)
MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX R

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

NONE

METHOD

DIAGONALIZATION METHOD ORIGINATED BY JACOBI AND ADAPTED
BY VON NEUMANN FOR LARGE COMPUTERS AS FOUND IN 'MATHEMATICAL
METHODS FOR DIGITAL COMPUTERS', EDITED BY A. RALSTON AND
H.S. GILF, JOHN WILEY AND SONS, NEW YORK, 1962, CHAPTER 7

SUBROUTINE EIGEN(A,R,N,MV)
DIMENSION A(1),R(1)

IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
STATEMENT WHICH FOLLOWS.

DOUBLE PRECISION A,R,ANORM,ANRXX,THR,X,Y,SINX,SINX2,COSX,
COSX2,SINCS,RANGE

THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
ROUTINE.

THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS. SORT IN STATEMENTS

EIGE 10
EIGE 20
EIGE 30
EIGE 40
EIGE 50
EIGE 60
EIGE 70
EIGE 80
EIGE 90
EIGE 100
EIGE 110
EIGE 120
EIGE 130
EIGE 140
EIGE 150
EIGE 160
EIGE 170
EIGE 180
EIGE 190
EIGE 200
EIGE 210
EIGE 220
EIGE 230
EIGE 240
EIGE 250
EIGE 260
EIGE 270
EIGE 280
EIGE 290
EIGE 300
EIGE 310
EIGE 320
EIGE 330
EIGE 340
EIGE 350
EIGE 360
EIGE 370
EIGE 380
EIGE 390
EIGE 400
EIGE 410
EIGE 420
EIGE 430
EIGE 440
EIGE 450
EIGE 460
EIGE 470
EIGE 480
EIGE 490
EIGE 500
EIGE 510
EIGE 520
EIGE 530
EIGE 540
EIGE 550
EIGE 560
EIGE 570
EIGE 580

```

MEMBER NAME EIGEN
40, 44, 75, AND 7A MUST BE CHANGED TO DSQRT. AHS IN STATEMENT
62 MUST BE CHANGED TO DAHS. THE CONSTANT IN STATEMENT 5 SHOULD
BE CHANGED TO 1.00-12.
.....
GENERATE IDENTITY MATRIX
5  RANGE=1.0E-6
  IF (N-1) 10,25,10
10  I=1
  DO 20 J=1,N
    IJ=I+N
    DO 20 I=1,N
      IJ=I+1
      R(IJ)=0.0
      IF (I-J) 20,15,20
15  R(IJ)=1.0
20  CONTINUE

      COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX)
25  ANORM=0.0
  DO 35 I=1,N
    DO 35 J=1,N
      IF (I-J) 30,35,30
30  IA=1+(J+J-J)/2
    ANORM=ANORM+A(IA)*A(IA)
35  CONTINUE
  IF (ANORM) 165,165,40
40  ANORMX=1.414*SQRT(ANORM)
  ANORMX=ANORMX/RANGE/FLOAT(N)

      INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR
      IND=0
      THR=ANORM
45  THR=THR/FLOAT(N)
50  L=1
55  V=L+1

      COMPUTE SIN AND COS
60  MQ=(M+M-M)/2
  LO=(L+L-L)/2
  LM=L+MQ
62  IF (ABS(A(LM))-THR) 130,65,65
65  IND=1
  LL=L+LO
  MM=M+MQ
  X=0.5*(A(LL)-A(MM))
68  Y=-A(L)/SQRT(A(LM)*A(LM)+X*X)
  IF (X) 70,75,75
70  Y=-Y
75  SINX=Y/SQRT(2.0*(1.0+(SQRT(1.0-Y*Y))))
  SINX2=SINX*SINX
7A  COSX=SQRT(1.0-SINX2)
  COSX2=COSX*COSX

```

```

EIGE 590
EIGE 600
EIGE 610
EIGE 620
EIGE 630
EIGE 640
EIGE 650
EIGE 660
EIGE 670
EIGE 680
EIGE 690
EIGE 700
EIGE 710
EIGE 720
EIGE 730
EIGE 740
EIGE 750
EIGE 760
EIGE 770
EIGE 780
EIGE 790
EIGE 800
EIGE 810
EIGE 820
EIGE 830
EIGE 840
EIGE 850
EIGE 860
EIGE 870
EIGE 880
EIGE 890
EIGE 900
EIGE 910
EIGE 920
EIGE 930
EIGE 940
EIGE 950
EIGE 960
EIGE 970
EIGE 980
EIGE 990
EIGE 1000
EIGE 1010
EIGE 1020
EIGE 1030
EIGE 1040
EIGE 1050
EIGE 1060
EIGE 1070
EIGE 1080
EIGE 1090
EIGE 1100
EIGE 1110
EIGE 1120
EIGE 1130
EIGE 1140
EIGE 1150
EIGE 1160

```

```

MEMBER NAME EIGEN
SINCS =SINX*COSX
C
C      ROTATE L AND M COLUMNS
C
      ILQ=0*(L-1)
      IMQ=0*(M-1)
      DO 125 I=1,N
      IQ=(I*1-1)/2
      IF(I-L) 80,115,80
      IF(I-M) 85,115,90
      80 IV=I+M
      85 IV=I+M
      90 IV=I+M
      95 IF(I-L) 100,105,105
      100 II=I+LW
      105 IL=L+IQ
      110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
      115 IF(MV-1) 120,125,120
      120 ILR=ILQ+I
      125 CONTINUE
      X=2.0*A(LM)*SINCS
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
C
C      TESTS FOR COMPLETION
C
C      TEST FOR M = LAST COLUMN
C
      130 IF(M-N) 135,140,135
      135 M=M+1
      GO TO 60
C
C      TEST FOR L = SECOND FROM LAST COLUMN
C
      140 IF(L-(N-1)) 145,150,145
      145 L=L+1
      GO TO 55
      150 IF(MV-1) 160,155,160
      155 MV=0
      GO TO 50
C
C      COMPARE THRESHOLD WITH FINAL NORM
C
      160 IF(THR-ANRMX) 165,165,45
C
C      SORT EIGENVALUES AND EIGENVECTORS
C
      165 IC=-N

```

```

EIGE1170
EIGE1180
EIGE1190
EIGE1200
EIGE1210
EIGE1220
EIGE1230
EIGE1240
EIGE1250
EIGE1260
EIGE1270
EIGE1280
EIGE1290
EIGE1300
EIGE1310
EIGE1320
EIGE1330
EIGE1340
EIGE1350
EIGE1360
EIGE1370
EIGE1380
EIGE1390
EIGE1400
EIGE1410
EIGE1420
EIGE1430
EIGE1440
EIGE1450
EIGE1460
EIGE1470
EIGE1480
EIGE1490
EIGE1500
EIGE1510
EIGE1520
EIGE1530
EIGE1540
EIGE1550
EIGE1560
EIGE1570
EIGE1580
EIGE1590
EIGE1600
EIGE1610
EIGE1620
EIGE1630
EIGE1640
EIGE1650
EIGE1660
EIGE1670
EIGE1680
EIGE1690
EIGE1700
EIGE1710
EIGE1720
EIGE1730
EIGE1740

```

```

MEMBER NAME EIGEN
DO 185 I=1,N
  IC=10+N
  LL=1+(1+I-1)/2
  JJ=N+(1-2)
  DO 185 J=1,N
    JJ=JJ+N
    MM=J+(J+J-J)/2
    IF (A(LL)-A(MM)) 170,185,185
170  X=A(LL)
    A(LL)=A(MM)
    A(MM)=X
    IF (MM-1) 175,185,175
175  DO 180 K=1,N
    ILK=10+K
    IMK=J0+K
    X=R(ILK)
    R(ILK)=R(IMK)
    R(IMK)=X
180  R(IMK)=X
185  CONTINUE
RETURN
END

```

```

EIGE1750
EIGE1760
EIGE1770
EIGE1780
EIGE1790
EIGE1800
EIGE1810
EIGE1820
EIGE1830
EIGE1840
EIGE1850
EIGE1860
EIGE1870
EIGE1880
EIGE1890
EIGE1900
EIGE1910
EIGE1920
EIGE1930
EIGE1940
EIGE1950

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MEMBER NAME DMFSD

```

.....
SUBROUTINE DMFSD
PURPOSE
  FACTOR A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX
USAGE
  CALL DMFSD(A,N,EPS,IER)
DESCRIPTION OF PARAMETERS
  A - DOUBLE PRECISION UPPER TRIANGULAR PART OF GIVEN
      SYMMETRIC POSITIVE DEFINITE N BY N COEFFICIENT
      MATRIX.
      ON RETURN A CONTAINS THE RESULTANT UPPER
      TRIANGULAR MATRIX IN DOUBLE PRECISION.
  N - THE NUMBER OF ROWS (COLUMNS) IN GIVEN MATRIX.
  EPS - SINGLE PRECISION INPUT CONSTANT WHICH IS USED
        AS RELATIVE TOLERANCE FOR TEST ON LOSS OF
        SIGNIFICANCE.
  IER - RESULTING ERROR PARAMETER CODED AS FOLLOWS
        IER=0 - NO ERROR
        IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAME-
        TER N OR BECAUSE SOME RADICAND IS NON-
        POSITIVE (MATRIX A IS NOT POSITIVE
        DEFINITE, POSSIBLY DUE TO LOSS OF SIGNI-
        FICANCE)
        IER=K - WARNING WHICH INDICATES LOSS OF SIGNIFI-
        CANCE. THE RADICAND FORMED AT FACTORIZA-
        TION STEP K+1 WAS STILL POSITIVE BUT NO
        LONGER GREATER THAN ABS(EPS*A(K+1,K+1)).
REMARKS
  THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE
  STORED COLUMNWISE IN N*(N+1)/2 SUCCESSIVE STORAGE LOCATIONS.
  IN THE SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGU-
  LAR MATRIX IS STORED COLUMNWISE TOO.
  THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL
  CALCULATED RADICANDS ARE POSITIVE.
  THE PRODUCT OF RETURNED DIAGONAL TERMS IS EQUAL TO THE
  SQUARE-ROOT OF THE DETERMINANT OF THE GIVEN MATRIX.
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
  NONE
METHOD
  SOLUTION IS DONE USING THE SQUARE-ROOT METHOD OF CHOLSKY.
  THE GIVEN MATRIX IS REPRESENTED AS PRODUCT OF TWO TRIANGULAR
  MATRICES, WHERE THE LEFT HAND FACTOR IS THE TRANSPOSE OF
  THE RETURNED RIGHT HAND FACTOR.
.....
SUBROUTINE DMFSD(A,N,EPS,IER)
DIMENSION A(1)

```

```

DMSD 10
DMSD 20
DMSD 30
DMSD 40
DMSD 50
DMSD 60
DMSD 70
DMSD 80
DMSD 90
DMSD 100
DMSD 110
DMSD 120
DMSD 130
DMSD 140
DMSD 150
DMSD 160
DMSD 170
DMSD 180
DMSD 190
DMSD 200
DMSD 210
DMSD 220
DMSD 230
DMSD 240
DMSD 250
DMSD 260
DMSD 270
DMSD 280
DMSD 290
DMSD 300
DMSD 310
DMSD 320
DMSD 330
DMSD 340
DMSD 350
DMSD 360
DMSD 370
DMSD 380
DMSD 390
DMSD 400
DMSD 410
DMSD 420
DMSD 430
DMSD 440
DMSD 450
DMSD 460
DMSD 470
DMSD 480
DMSD 490
DMSD 500
DMSD 510
DMSD 520
DMSD 530
DMSD 540
DMSD 550
DMSD 560
DMSD 570
DMSD 580

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MEMBER NAME DMSD
DOUBLE PRECISION DPIV,DSUM,A
C
C      TEST ON WRONG INPUT PARAMETER N
C      IF(N=1) 12,1,1
1  IER=0
C
C      INITIALIZE DIAGONAL-LOOP
C      KPIV=0
C      DO 11 K=1,N
C      KPIV=KPIV+K
C      IND=KPIV
C      LEND=K-1
C
C      CALCULATE TOLERANCE
C      TOL=A*S(EPS*SNGL(A(KPIV)))
C
C      START FACTORIZATION-LOOP OVER K-TH ROW
C      DO 11 I=K,N
C      DSUM=0.00
C      IF(LEND) 2,4,2
C
C      START INNER LOOP
C      2 DO 3 L=1,LEND
C      LANF=KPIV-L
C      LIND=IND-L
C      3 DSUM=DSUM+A(LANF)*A(LIND)
C      END OF INNER LOOP
C
C      TRANSFORM ELEMENT A(IND)
C      4 DSUM=A(IND)-DSUM
C      IF(I=K) 10,5,10
C
C      TEST FOR NEGATIVE PIVOT ELEMENT AND FOR LOSS OF SIGNIFICANCE
C      5 IF(SNGL(DSUM)-TOL) 6,6,9
C      6 IF(DSUM) 12,12,7
C      7 IF(ITER) 8,8,9
C      8 IER=K-1
C
C      COMPUTE PIVOT ELEMENT
C      9 DPIV=DSORT(DSUM)
C      A(KPIV)=DPIV
C      DPIV=1.00/DPIV
C      GO TO 11
C
C      CALCULATE TERMS IN ROW
C      10 A(IND)=DSUM*DPIV
C      11 IND=IND+1
C      END OF DIAGONAL-LOOP
C
C      RETURN
C      12 IER=-1
C      RETURN
C      END

```

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DMSD 590
DMSD 600
DMSD 610
DMSD 620
DMSD 630
DMSD 640
DMSD 650
DMSD 660
DMSD 670
DMSD 680
DMSD 690
DMSD 700
DMSD 710
DMSD 720
DMSD 730
DMSD 740
DMSD 750
DMSD 760
DMSD 770
DMSD 780
DMSD 790
DMSD 800
DMSD 810
DMSD 820
DMSD 830
DMSD 840
DMSD 850
DMSD 860
DMSD 870
DMSD 880
DMSD 890
DMSD 900
DMSD 910
DMSD 920
DMSD 930
DMSD 940
DMSD 950
DMSD 960
DMSD 970
DMSD 980
DMSD 990
DMSD1000
DMSD1010
DMSD1020
DMSD1030
DMSD1040
DMSD1050
DMSD1060
DMSD1070
DMSD1080
DMSD1090
DMSD1100
DMSD1110

```

MEMBER NAME DSINV

SUBROUTINE DSINV

PURPOSE

INVERT A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX

USAGE

CALL DSINV(A,N,EPS,IER)

DESCRIPTION OF PARAMETERS

A - DOUBLE PRECISION UPPER TRIANGULAR PART OF GIVEN SYMMETRIC POSITIVE DEFINITE N BY N COEFFICIENT MATRIX.

N - THE NUMBER OF ROWS (COLUMNS) IN GIVEN MATRIX.

EPS - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS RELATIVE TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.

IER - RESULTING ERROR PARAMETER CODED AS FOLLOWS

IER=0 - NO ERROR

IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAMETER N OR BECAUSE SOME RADICAND IS NON-

POSITIVE (MATRIX A IS NOT POSITIVE DEFINITE, POSSIBLY DUE TO LOSS OF SIGNIFICANCE)

IER=K - WARNING WHICH INDICATES LOSS OF SIGNIFICANCE. THE RADICAND FORMED AT FACTORIZATION STEP K+1 WAS STILL POSITIVE BUT NO LONGER GREATER THAN $\text{ABS}(\text{EPS} \cdot \text{A}(K+1, K+1))$.

REMARKS

THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE STORED COLUMNWISE IN $N \cdot (N+1)/2$ SUCCESSIVE STORAGE LOCATIONS. IN THE SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGULAR MATRIX IS STORED COLUMNWISE TOO. THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL CALCULATED RADICANDS ARE POSITIVE.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
DMFSD

METHOD

SOLUTION IS DONE USING FACTORIZATION BY SUBROUTINE DMFSD.

SUBROUTINE DSINV(A,N,EPS,IER)

DIMENSION A(1)

DOUBLE PRECISION A,DIN,WORK

FACTORIZE GIVEN MATRIX BY MEANS OF SUBROUTINE DMFSD

A = TRANSPOSE(T) * T

CALL DMFSD(A,N,EPS,IER)

DSIN 10
DSIN 20
DSIN 30
DSIN 40
DSIN 50
DSIN 60
DSIN 70
DSIN 80
DSIN 90
DSIN 100
DSIN 110
DSIN 120
DSIN 130
DSIN 140
DSIN 150
DSIN 160
DSIN 170
DSIN 180
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DSIN 470
DSIN 480
DSIN 490
DSIN 500
DSIN 510
DSIN 520
DSIN 530
DSIN 540
DSIN 550
DSIN 560
DSIN 570
DSIN 580

```

MEMBER NAME DSINV
      IF(IER) 9,1,1
C
C      INVERT UPPER TRIANGULAR MATRIX T
C      PREPARE INVERSION-LOOP
1  IPIV=N*(N+1)/2
   IND=IPIV
C
C      INITIALIZE INVERSION-LOOP
   DO 4 I=1,N
     DIN=1.00/A(IPIV)
     A(IPIV)=0IN
     MIN=I
     KEND=I-1
     LEND=I-KEND
     IF(KEND) 5,5,2
2  J=IND
C
C      INITIALIZE ROW-LOOP
   DO 4 K=1,KEND
     WORK=0.00
     MIN=MIN-1
     LHOR=IPIV
     LVER=J
C
C      START INNER LOOP
   DO 3 L=LEND,MIN
     LVER=LVER+1
     LHOR=LHOR+L
3  WORK=WORK+A(LVER)*A(LHOR)
     END OF INNER LOOP
C
C      A(J)=-WORK*DIN
4  J=J-MIN
     END OF ROW-LOOP
5  IPIV=IPIV-MIN
6  IND=IND-1
     END OF INVERSION-LOOP
C
C      CALCULATE INVERSE(A) BY MEANS OF INVERSE(T)
C      INVERSE(A) = INVERSE(T) * TRANSPOSE(INVERSE(T))
C      INITIALIZE MULTIPLICATION-LOOP
   DO 7 I=1,N
     IPIV=IPIV+1
     J=IPIV
C
C      INITIALIZE ROW-LOOP
   DO 7 K=1,N
     WORK=0.00
     LHOR=J
C
C      START INNER LOOP
   DO 7 L=K,N
     LVER=LHOR+K-1
     WORK=WORK+A(LHOR)*A(LVER)
7  LHOR=LHOR+L
     END OF INNER LOOP

```

```

DSIN 590
DSIN 600
DSIN 610
DSIN 620
DSIN 630
DSIN 640
DSIN 650
DSIN 660
DSIN 670
DSIN 680
DSIN 690
DSIN 700
DSIN 710
DSIN 720
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DSIN 980
DSIN 990
DSIN1000
DSIN1010
DSIN1020
DSIN1030
DSIN1040
DSIN1050
DSIN1060
DSIN1070
DSIN1080
DSIN1090
DSIN1100
DSIN1110
DSIN1120
DSIN1130
DSIN1140
DSIN1150
DSIN1160

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```

MEMBER NAME DSINV
      A(J)=B*DKK
      8 J=J+K
      C      END OF ROW- AND MULTIPLICATION-LOOP
      C
      9 RETURN
      END

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DSIN1170
DSIN1180
DSIN1190
DSIN1200
DSIN1210
DSIN1220

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